

pomegranate

fast and flexible probabilistic modeling in python

Maxwell Libbrecht

Postdoc, Department of Genome Sciences,
University of Washington, Seattle WA
Assistant Professor, Simon Fraser University,
Vancouver BC

noble.gs.washington.edu/~maxwl/

Jacob Schreiber

PhD student, Paul G. Allen School of
Computer Science, University of
Washington, Seattle WA



[jmschreiber91](#)



[@jmschrei](#)



[@jmschreiber91](#)

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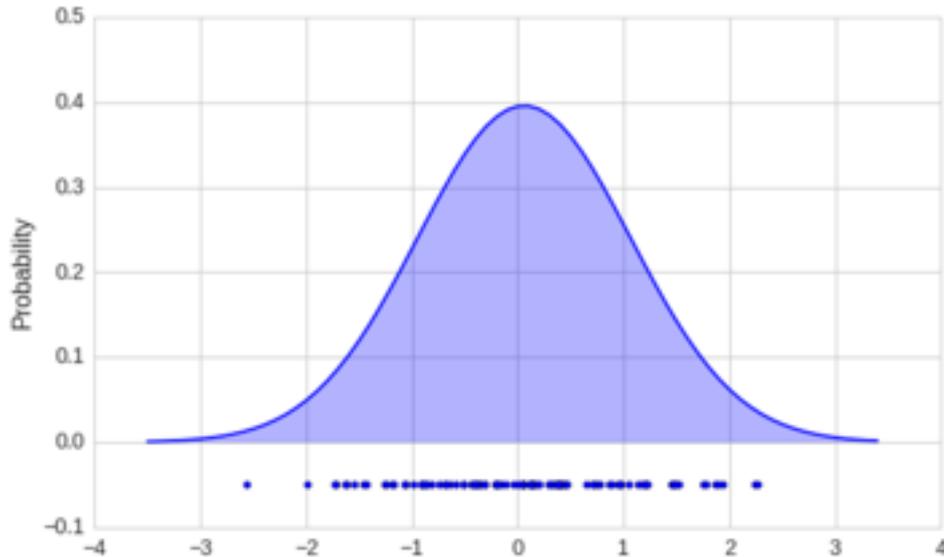


Follow along with the Jupyter tutorial

goo.gl/cbE7rs



What is probabilistic modeling?



Tasks:

- Sample from a distribution.
- Learn parameters of a distribution from data.
- Predict which distribution generated a data example.



When should you use pomegranate?

Use scipy or scikit-learn

- logistic regression
- sample from a Gaussian distribution
- speed is not important

Use pomegranate

Use custom packages (in R or stand-alone)

- distributions not implemented in pomegranate
- problem-specific inference algorithms

simple

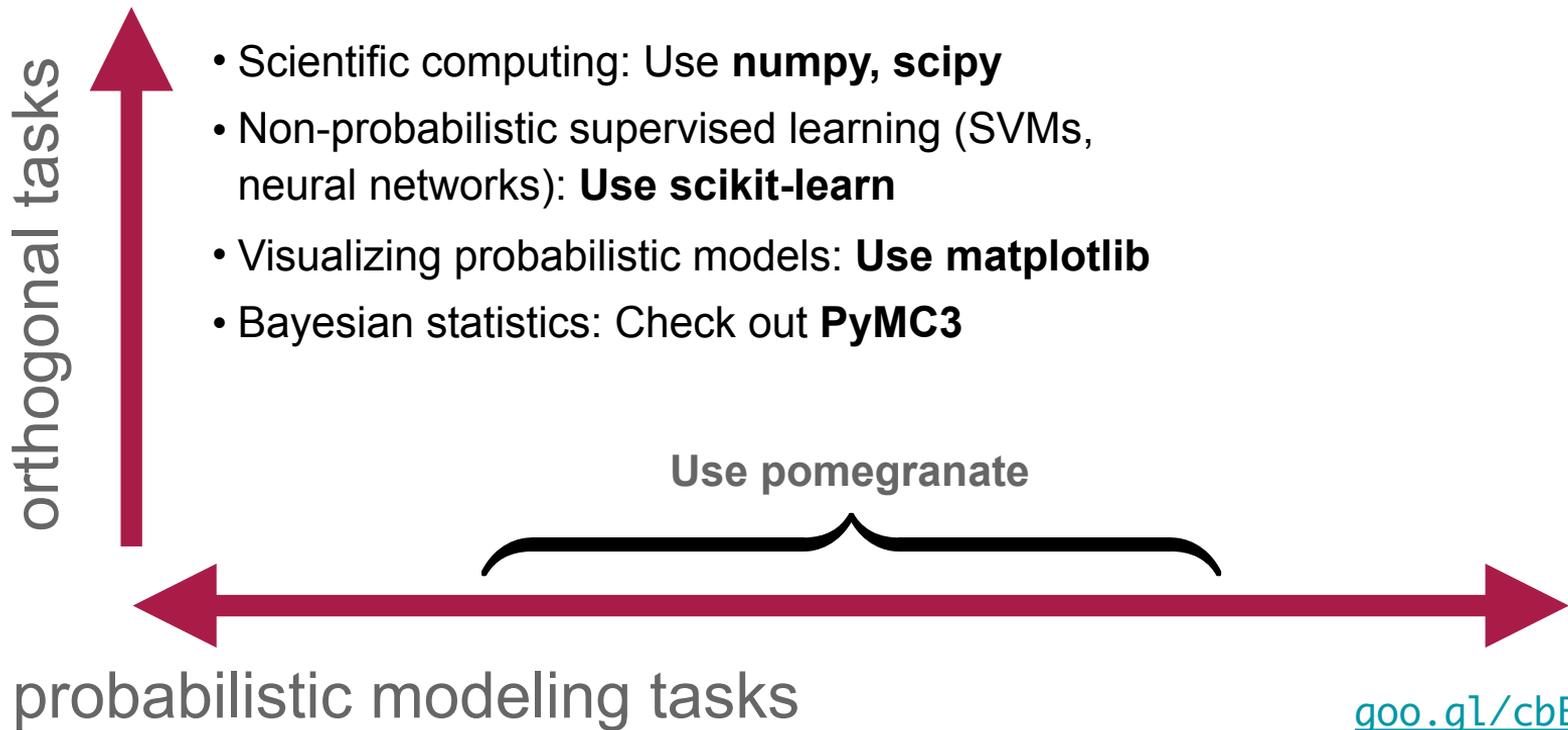
complex



complexity of probabilistic modeling task



When should you use pomegranate?





Overview: this talk

Overview

Major Models/Model Stacks

1. General Mixture Models
2. Hidden Markov Models
3. Bayesian Networks
4. Bayes Classifiers

Finale: Train a mixture of HMMs in parallel



The API is common to all models

`model.log_probability(X) / model.probability(X)`

`model.sample()`

`model.fit(X, weights, inertia)`

`model.summarize(X, weights)`

`model.from_summaries(inertia)`

`model.predict(X)`

`model.predict_proba(X)`

`model.predict_log_proba(X)`

`Model.from_samples(X, weights)`

All models have these methods!

All models composed of distributions (like GMM, HMM...) have these methods too!



Overview: supported models

Six Main Models:

1. Probability Distributions
2. General Mixture Models
3. Markov Chains
4. Hidden Markov Models
5. Bayes Classifiers / Naive Bayes
6. Bayesian Networks

Two Helper Models:

1. k-means++/kmeans||
2. Factor Graphs



pomegranate supports many distributions

Univariate Distributions

1. UniformDistribution
2. BernoulliDistribution
3. NormalDistribution
4. LogNormalDistribution
5. ExponentialDistribution
6. BetaDistribution
7. GammaDistribution
8. DiscreteDistribution
9. PoissonDistribution

Kernel Densities

1. GaussianKernelDensity
2. UniformKernelDensity
3. TriangleKernelDensity

Multivariate Distributions

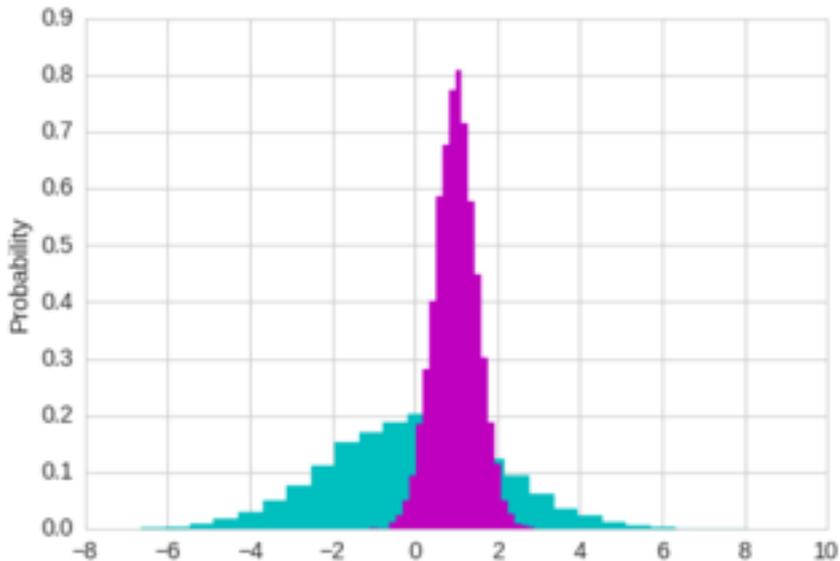
1. IndependentComponentsDistribution
2. MultivariateGaussianDistribution
3. DirichletDistribution
4. ConditionalProbabilityTable
5. JointProbabilityTable



Models can be created from known values

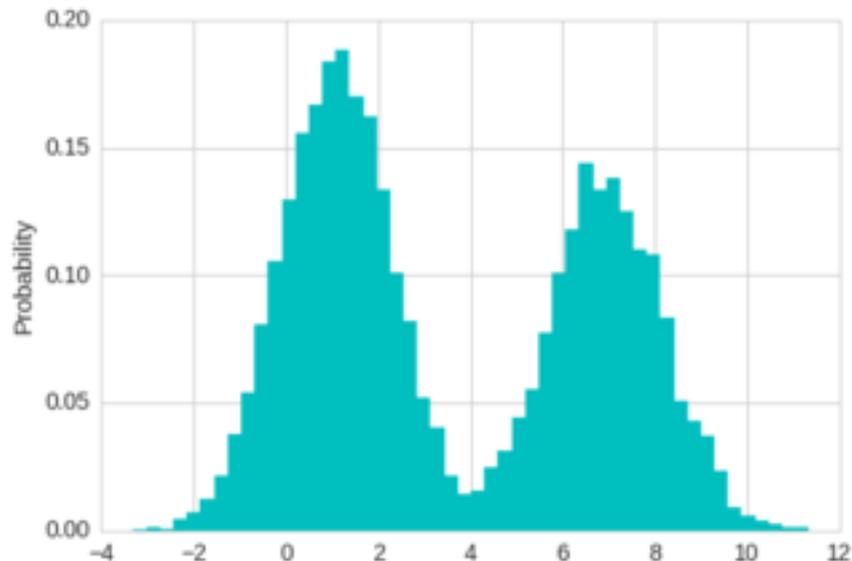
$\mu, \text{sig} = 0, 2$

`a = NormalDistribution(μ, sig)`



`X = [0, 1, 1, 2, 1.5, 6, 7, 8, 7]`

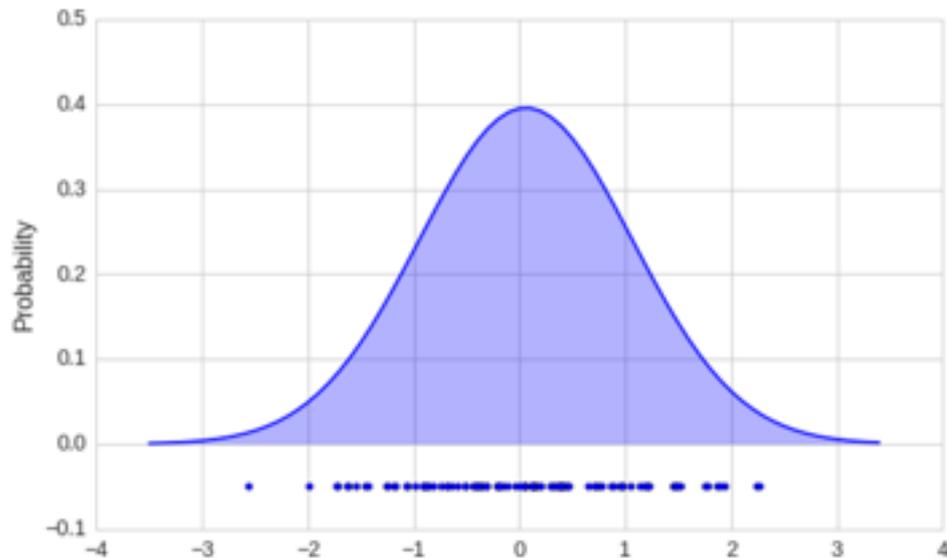
`a = GaussianKernelDensity(X)`





Models can be learned from data

```
X = numpy.random.normal(0, 1, 100)  
a = NormalDistribution.from_samples(X)
```

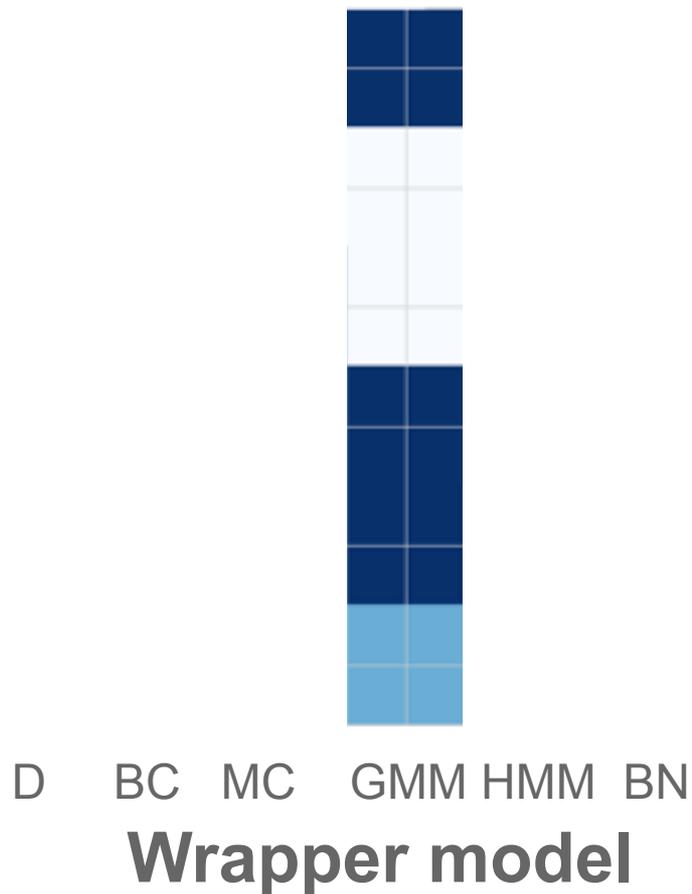




Overview: model stacking in pomegranate

Sub-model

- Distributions
- Bayes Classifiers
- Markov Chains
- General Mixture Models
- Hidden Markov Models
- Bayesian Networks





Overview: model stacking in pomegranate

Sub-model

Distributions

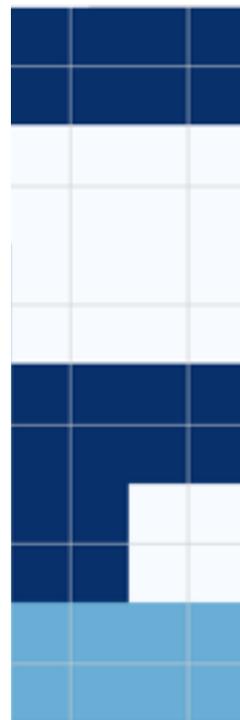
Bayes Classifiers

Markov Chains

General Mixture Models

Hidden Markov Models

Bayesian Networks



D BC MC GMM HMM BN

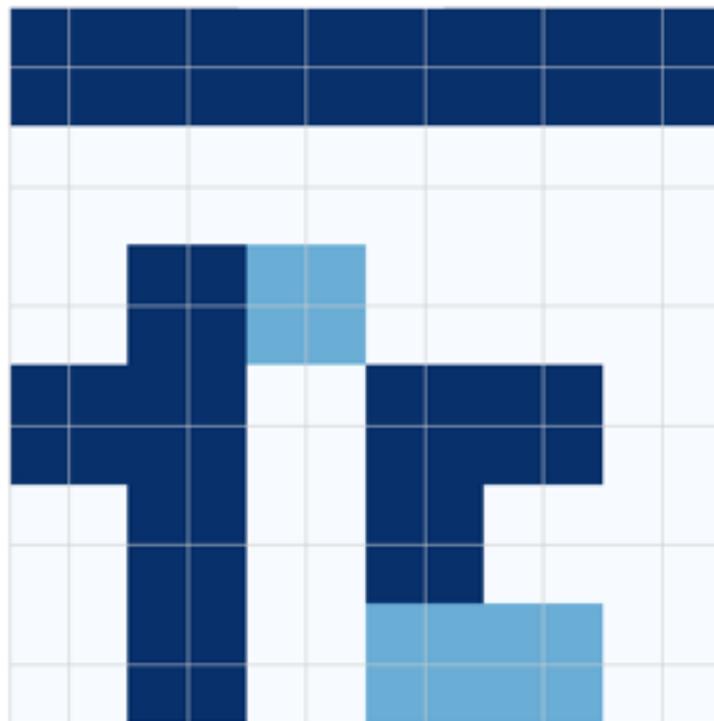
Wrapper model



Overview: model stacking in pomegranate

Sub-model

Distributions
Bayes Classifiers
Markov Chains
General Mixture Models
Hidden Markov Models
Bayesian Networks



D BC MC GMM HMM BN

Wrapper model



Overview: model stacking in pomegranate

Distributions

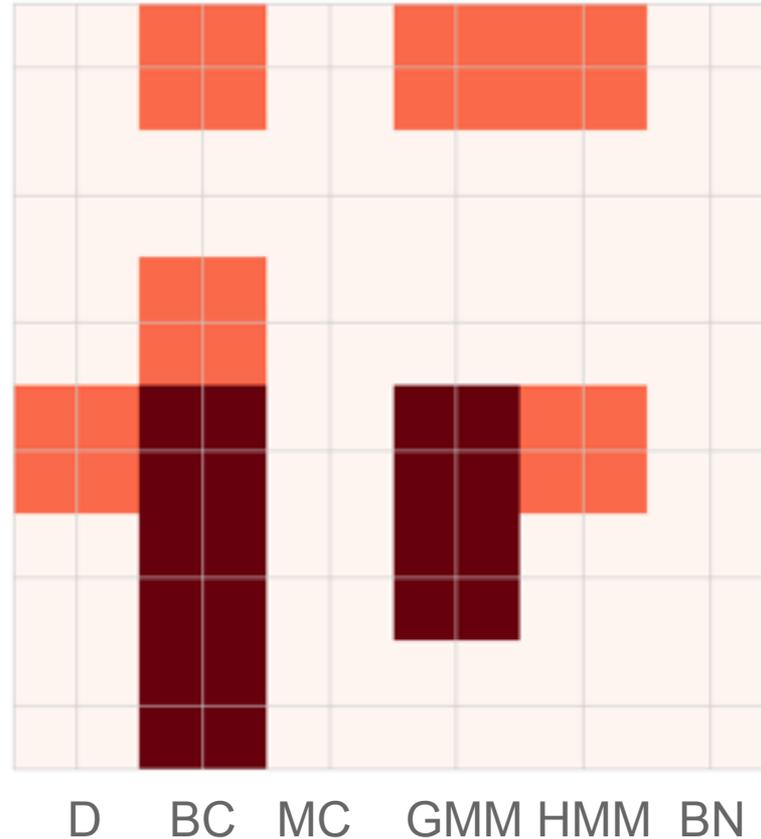
Bayes Classifiers

Markov Chains

General Mixture Models

Hidden Markov Models

Bayesian Networks





pomegranate can be faster than numpy

Fitting a Normal Distribution to 1,000 samples

```
data = numpy.random.randn(1000)

print "numpy time:"
%timeit -n 100 data.mean(), data.std()
print
print "pomegranate time:"
%timeit -n 100 NormalDistribution.from_samples(data)
```

numpy time:
100 loops, best of 3: 46.6 μ s per loop

pomegranate time:
100 loops, best of 3: 22.2 μ s per loop



pomegranate can be faster than numpy

Fitting Multivariate Gaussian to 10,000,000 samples of 10 dimensions

```
data = numpy.random.randn(10000000, 10)

print "numpy time:"
%timeit -n 10 data.mean(), numpy.cov(data.T)
print
print "pomegranate time:"
%timeit -n 10 MultivariateGaussianDistribution.from_samples(data)
```

numpy time:
10 loops, best of 3: 1.02 s per loop

pomegranate time:
10 loops, best of 3: 799 ms per loop



pomegranate uses BLAS internally

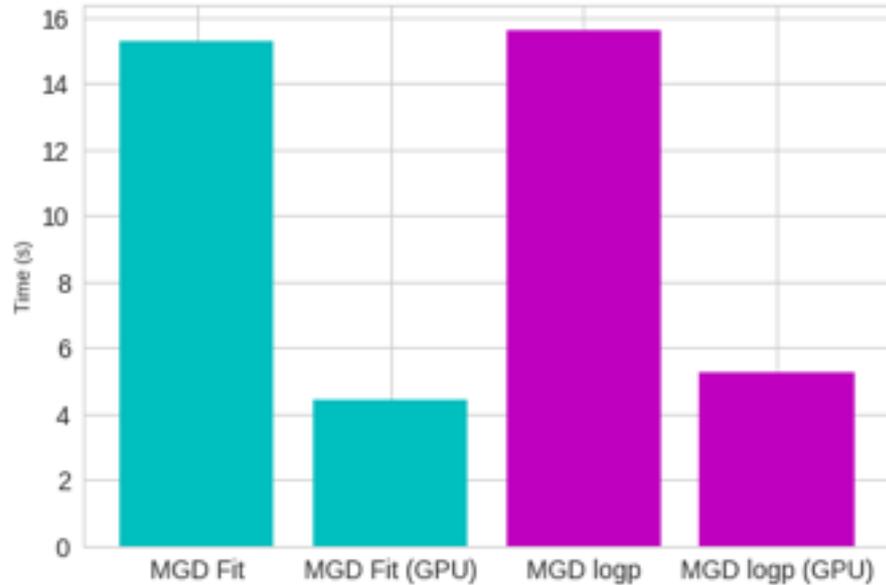
```
from scipy.linalg.cython_blas cimport dgemm
```

```
dgemm('N', 'T', &d, &d, &n, &alpha, y, &d, X, &d, &beta, pair_sum, &d)
```

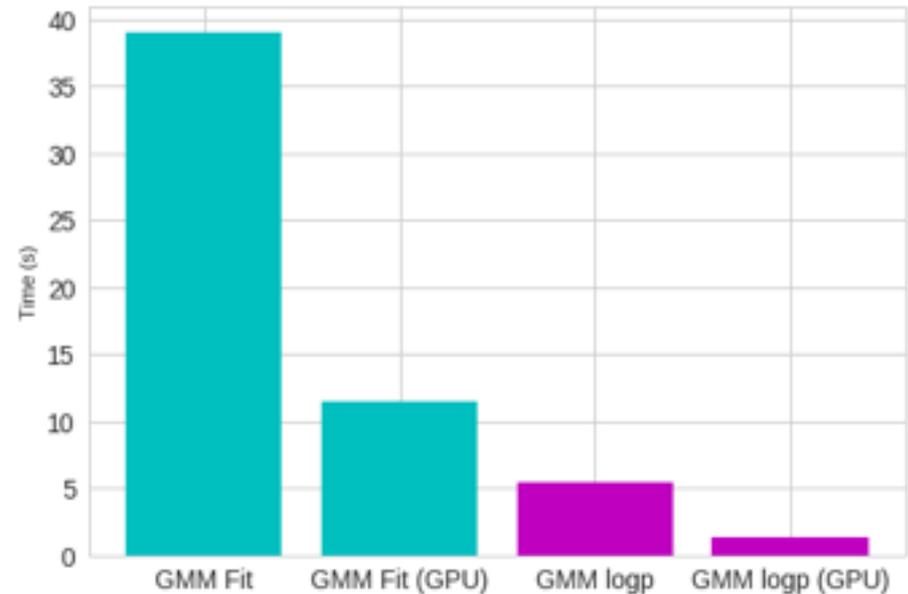


pomegranate just merged GPU support

Multivariate Gaussian with GPU Acceleration



Gaussian Mixture Model with GPU Acceleration





pomegranate uses additive summarization

pomegranate reduces data to sufficient statistics for updates and so only has to go datasets once (for all models).

Here is an example of the Normal Distribution sufficient statistics

$$\sum_{i=1}^n w_i \quad \sum_{i=1}^n w_i x_i \quad \sum_{i=1}^n w_i x_i^2$$



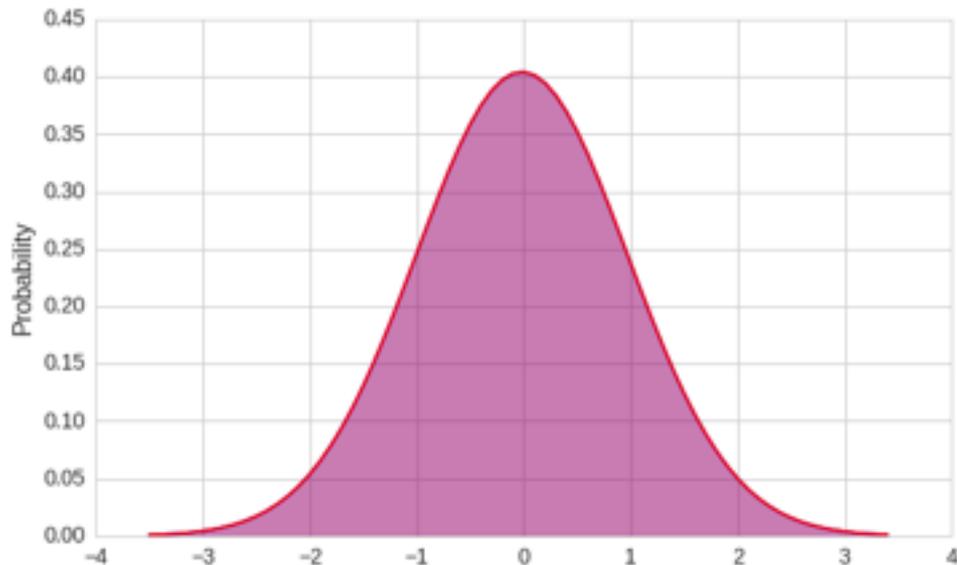
$$\mu = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$$
$$\sigma^2 = \frac{\sum_{i=1}^n w_i x_i^2}{\sum_{i=1}^n w_i} - \frac{\left(\sum_{i=1}^n w_i x_i\right)^2}{\left(\sum_{i=1}^n w_i\right)^2}$$



pomegranate supports out-of-core learning

Batches from a dataset can be reduced to additive summary statistics, enabling exact updates from data that can't fit in memory.

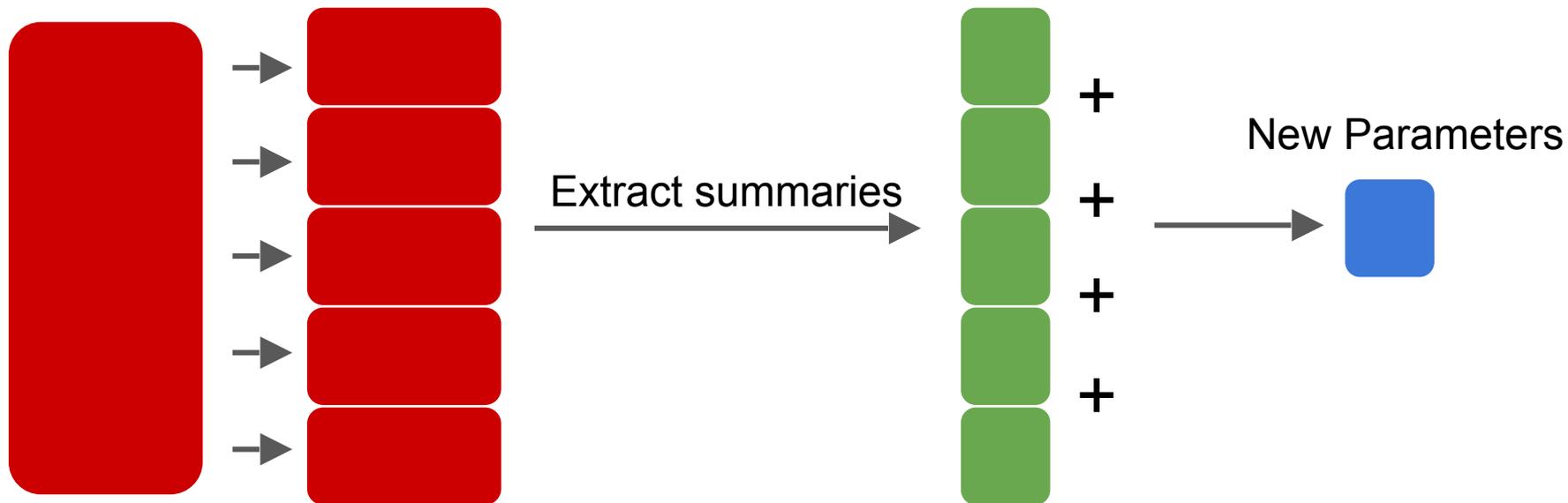
```
a.fit(data)
b.summarize(data[:1000])
b.summarize(data[1000:2000])
b.summarize(data[2000:3000])
b.summarize(data[3000:4000])
b.summarize(data[4000:])
b.from_summaries()
```



Fit Mean: -0.0174820965846, Fit STD: 0.986767322871
Summarize Mean: -0.0174820965846, Summarize STD: 0.986767322871



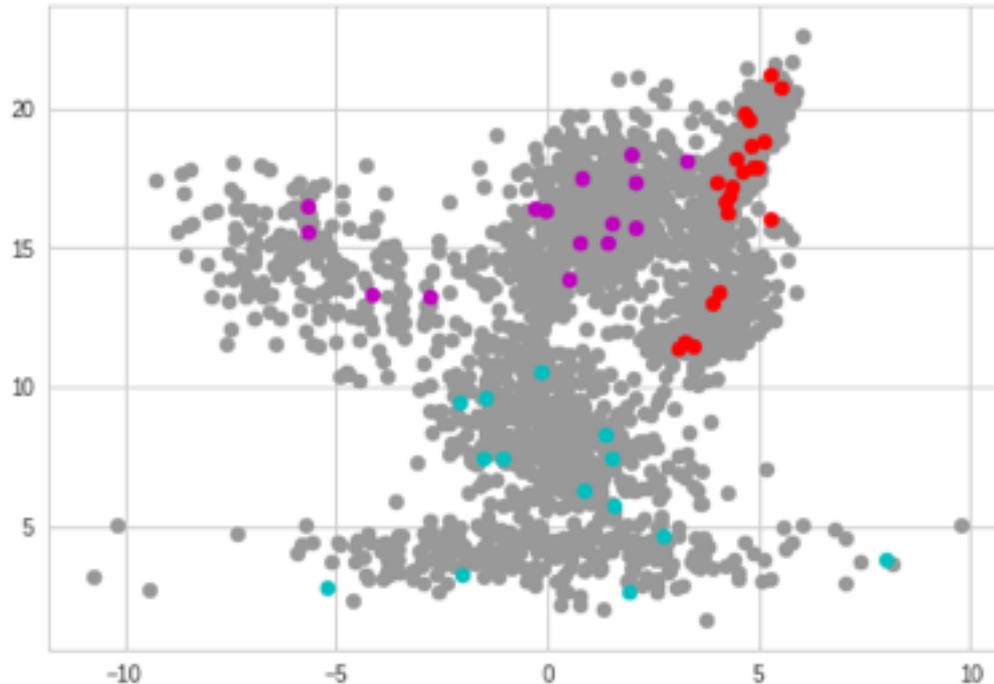
Parallelization exploits additive summaries





pomegranate supports semisupervised learning

Summary statistics from supervised models can be added to summary statistics from unsupervised models to train a single model on a mixture of labeled and unlabeled data.

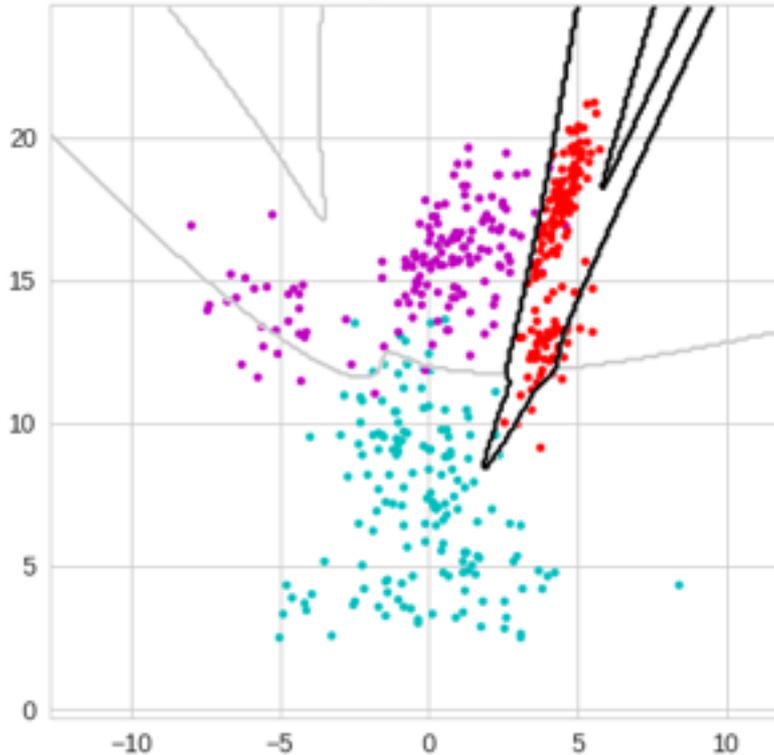




pomegranate supports semisupervised learning

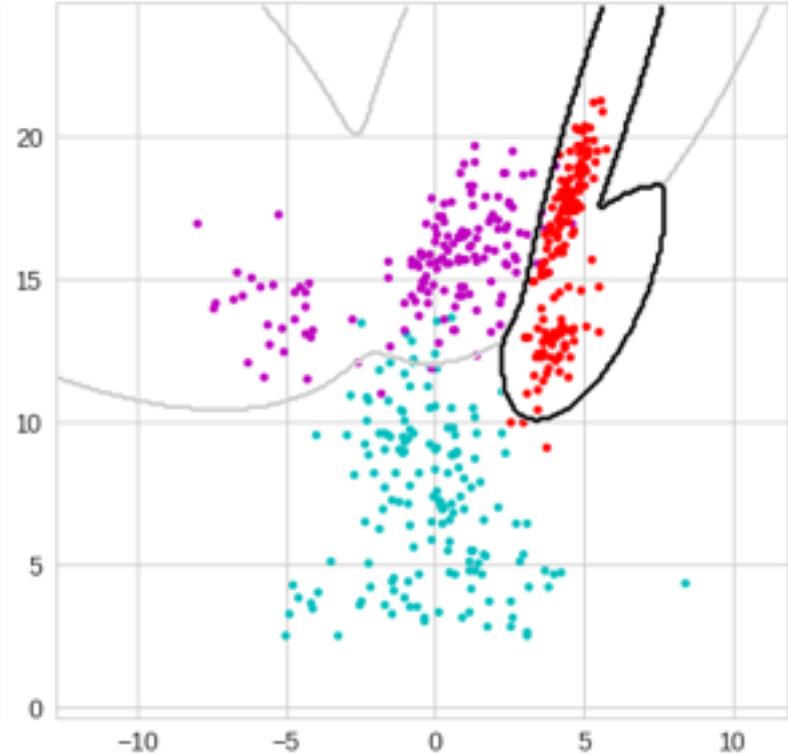
Supervised Accuracy: 0.93

Test Data, Supervised Boundaries



Semisupervised Accuracy: 0.96

Test Data, Semi-supervised Boundaries





pomegranate can be faster than scipy

```
mu, cov = numpy.random.randn(2000), numpy.eye(2000)
d = MultivariateGaussianDistribution(mu, cov)
X = numpy.random.randn(2000, 2000)
print "scipy time: ",
%timeit multivariate_normal.logpdf(X, mu, cov)
print "pomegranate time: ",
%timeit MultivariateGaussianDistribution(mu, cov).log_probability(X)
print "pomegranate time (w/ precreated object): ",
%timeit d.log_probability(X)
```

```
scipy time: 1 loop, best of 3: 1.67 s per loop
pomegranate time: 1 loop, best of 3: 801 ms per loop
pomegranate time (w/ precreated object): 1 loop, best of 3: 216 ms per loop
```



pomegranate uses aggressive caching

$$P(X|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

$$\log P(X|\mu, \sigma) = -\log(\sqrt{2\pi}\sigma) - \frac{(x - \mu)^2}{2\sigma^2}$$

$$\log P(X|\mu, \sigma) = \alpha - \frac{(x - \mu)^2}{\beta}$$



GOSSIP GIRL



Example 'blast' from Gossip Girl

Spotted: Lonely Boy. Can't believe the love of his life has returned. If only she knew who he was. But everyone knows Serena. And everyone is talking. Wonder what Blair Waldorf thinks. Sure, they're BFF's, but we always thought Blair's boyfriend Nate had a thing for Serena.



How do we encode these 'blasts'?

Better lock it down with Nate, B. Clock's ticking.

+1 Nate

-1 Blair



How do we encode these 'blasts'?

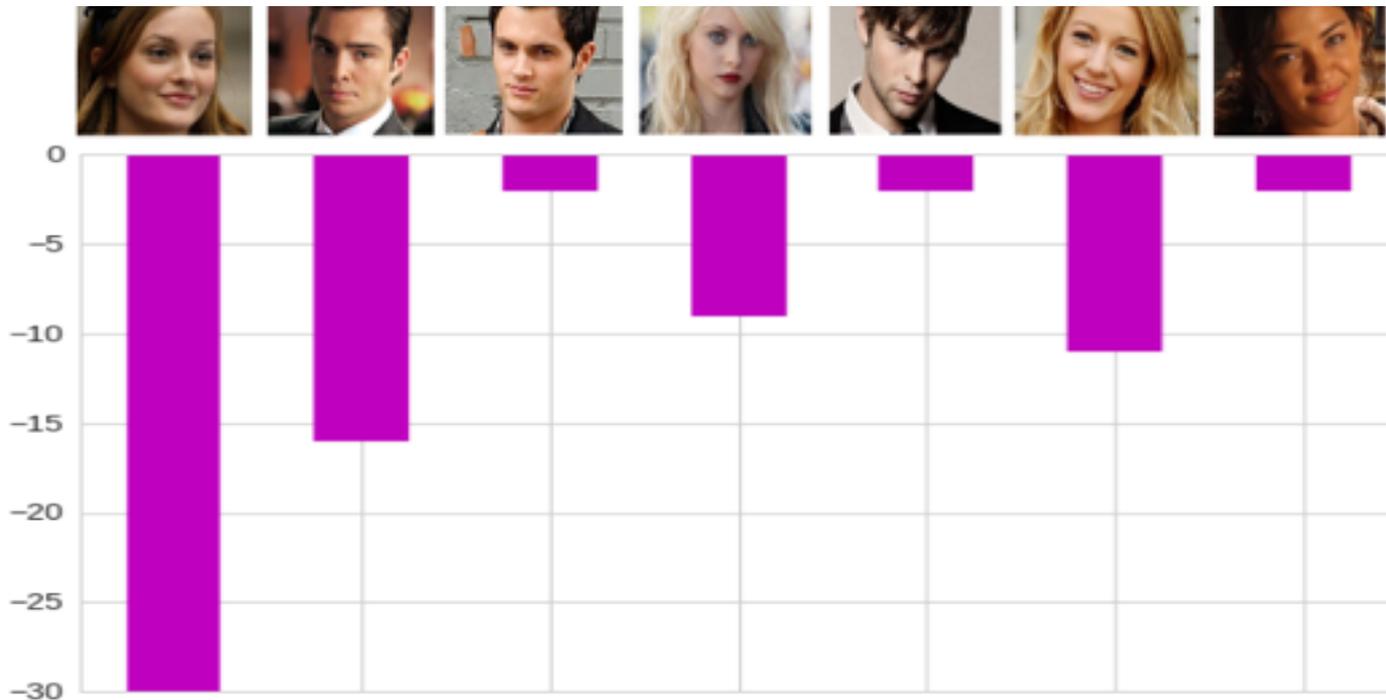
This just in: S and B committing a crime of fashion. Who doesn't love a five-finger discount. Especially if it's the middle one.

-1 Blair

-1 Serena

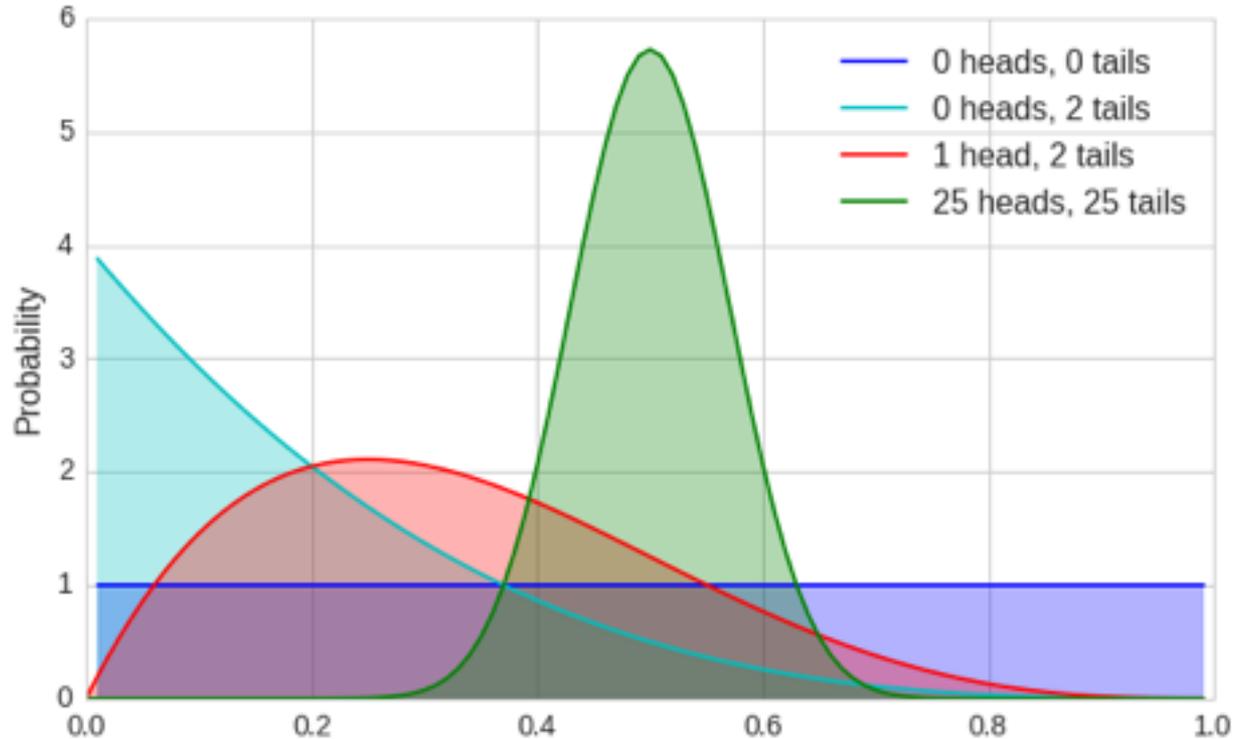


Simple summations don't work well



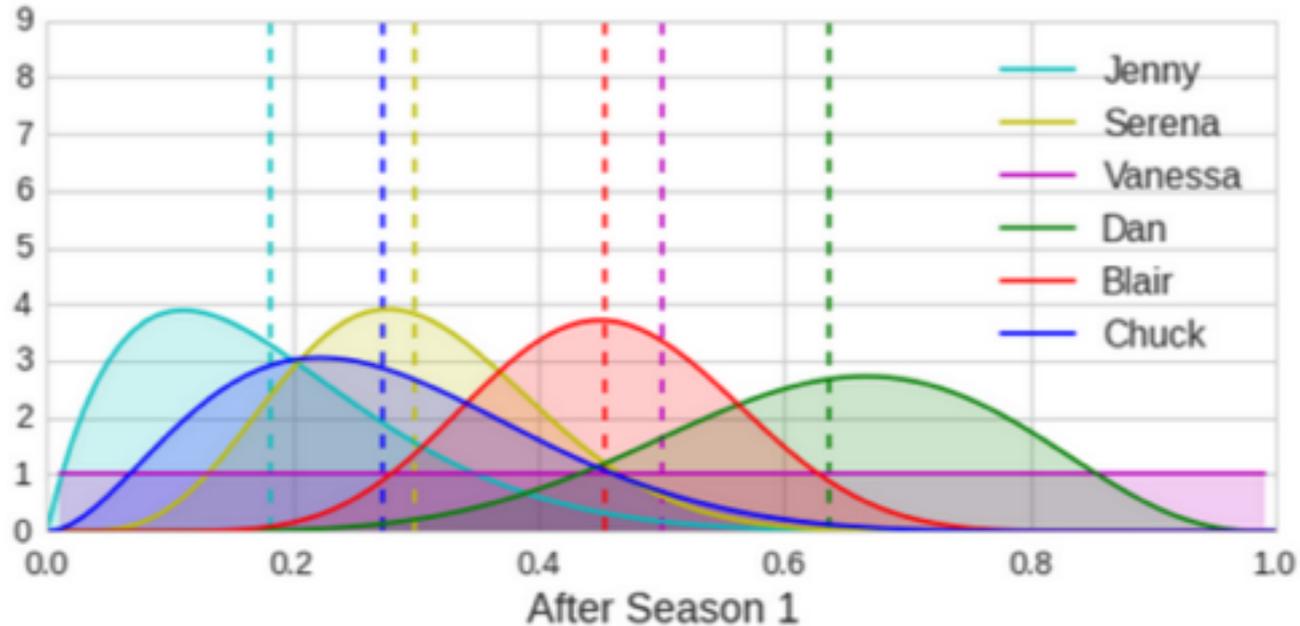


Beta distributions can model uncertainty



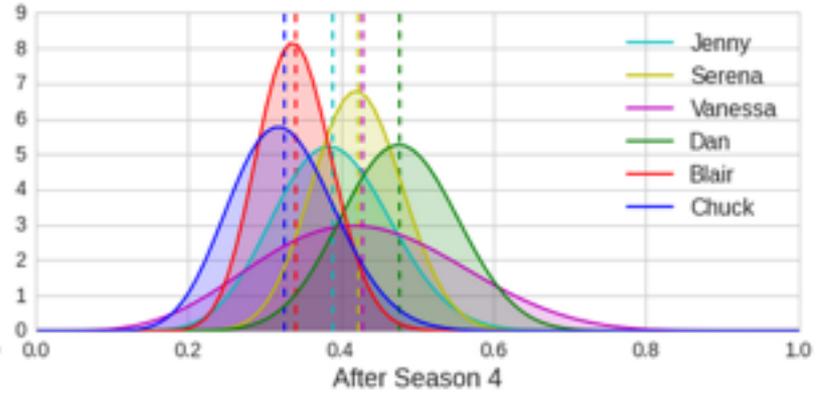
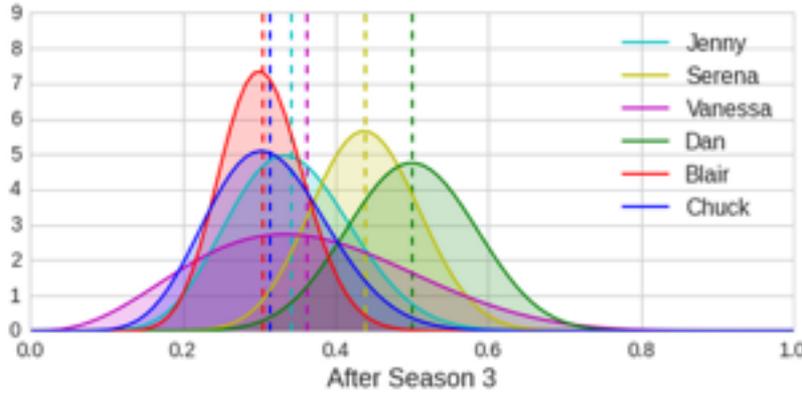
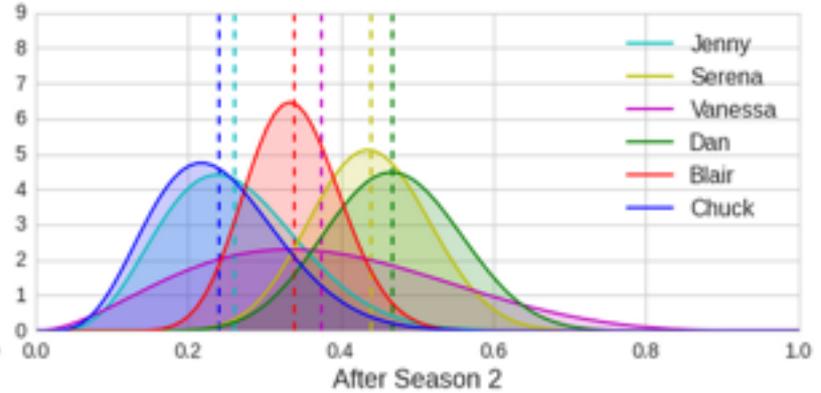
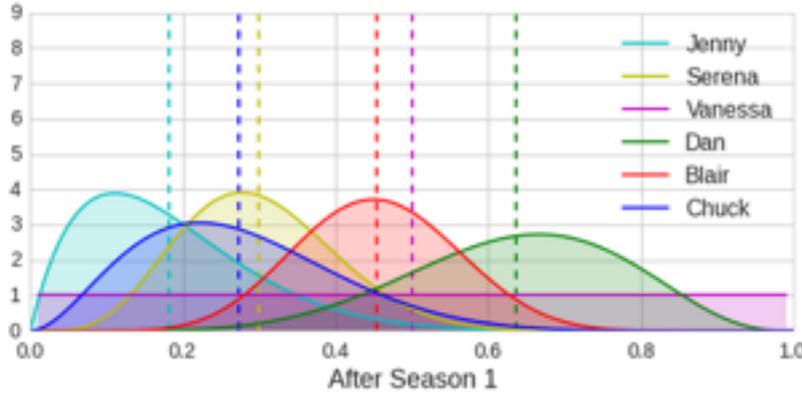


Beta distributions can model uncertainty





Beta distributions can model uncertainty





Overview: this talk

Overview

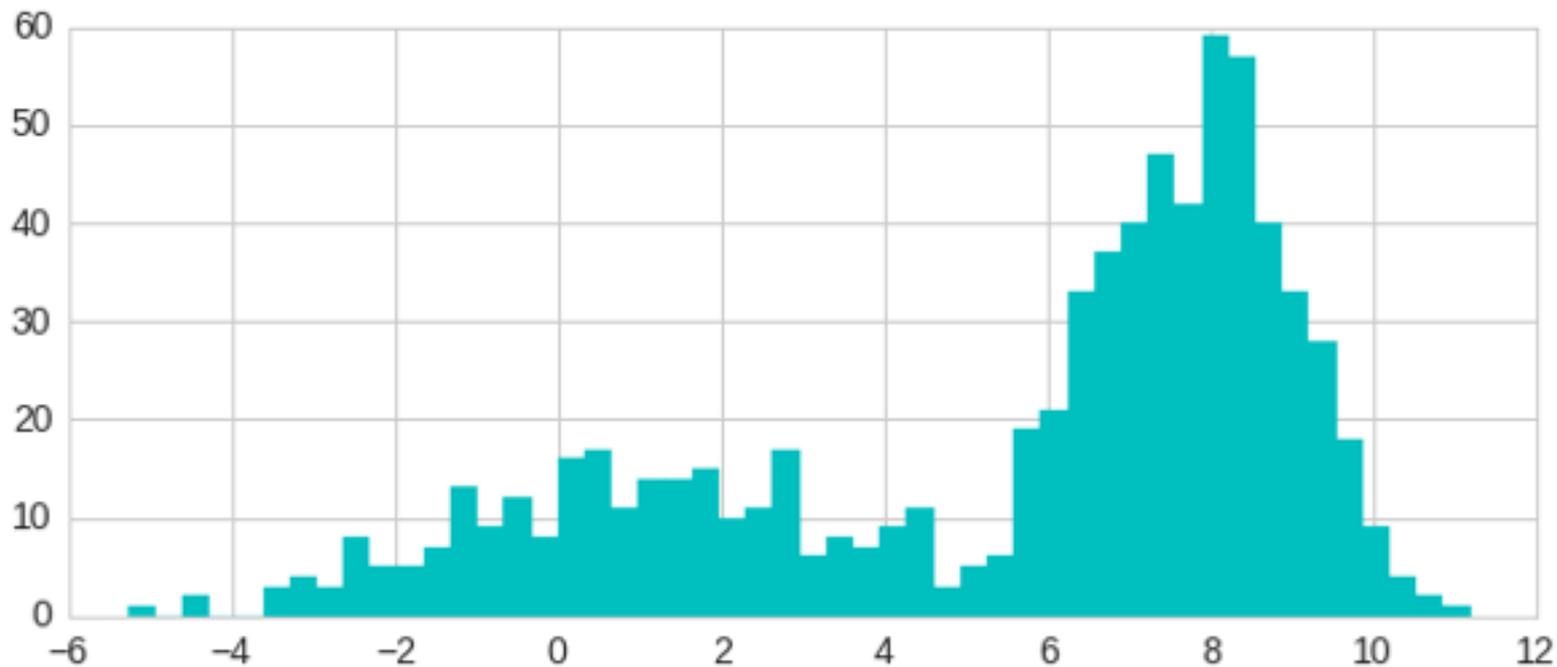
Major Models/Model Stacks

1. **General Mixture Models**
2. Hidden Markov Models
3. Bayesian Networks
4. Bayes Classifiers

Finale: Train a mixture of HMMs in parallel

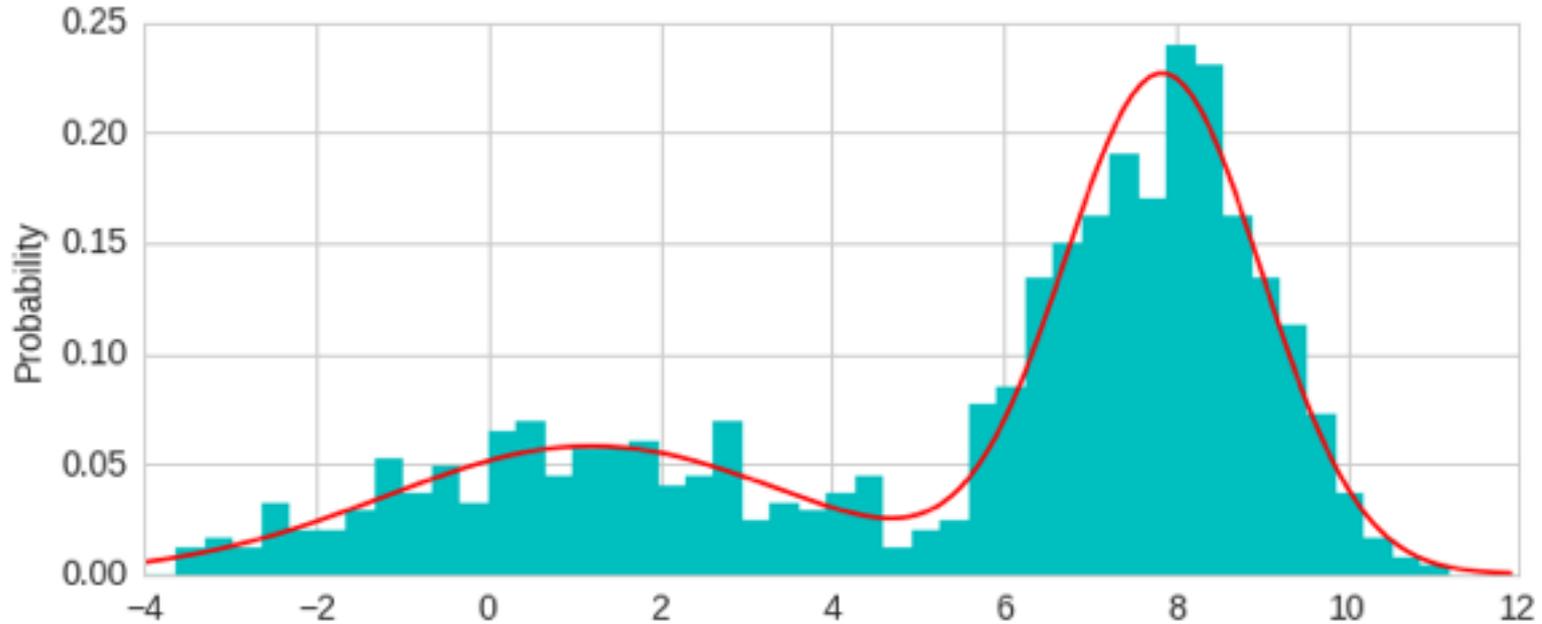


GMMs can model complex distributions





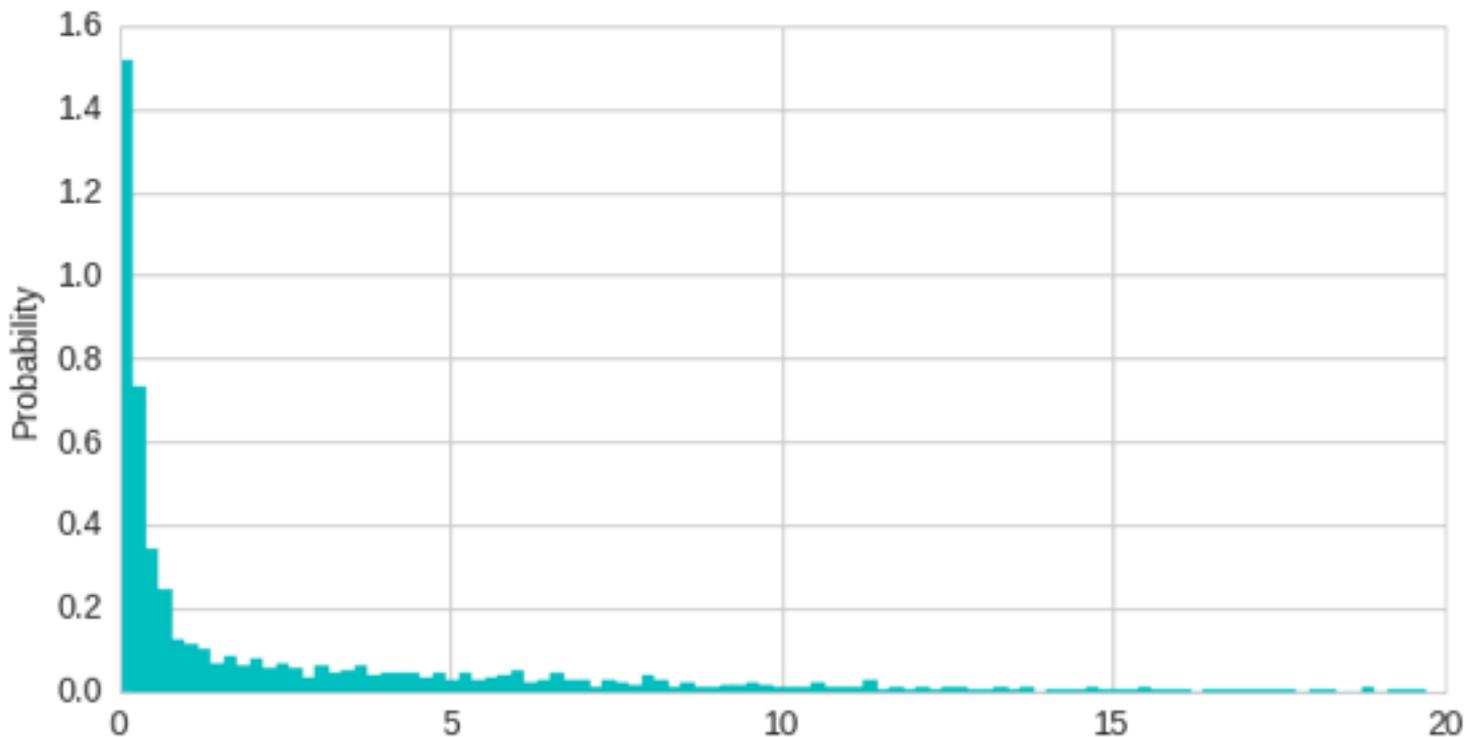
GMMs can model complex distributions



```
model = GeneralMixtureModel.from_samples(NormalDistribution, 2, X)
```



GMMs can model complex distributions





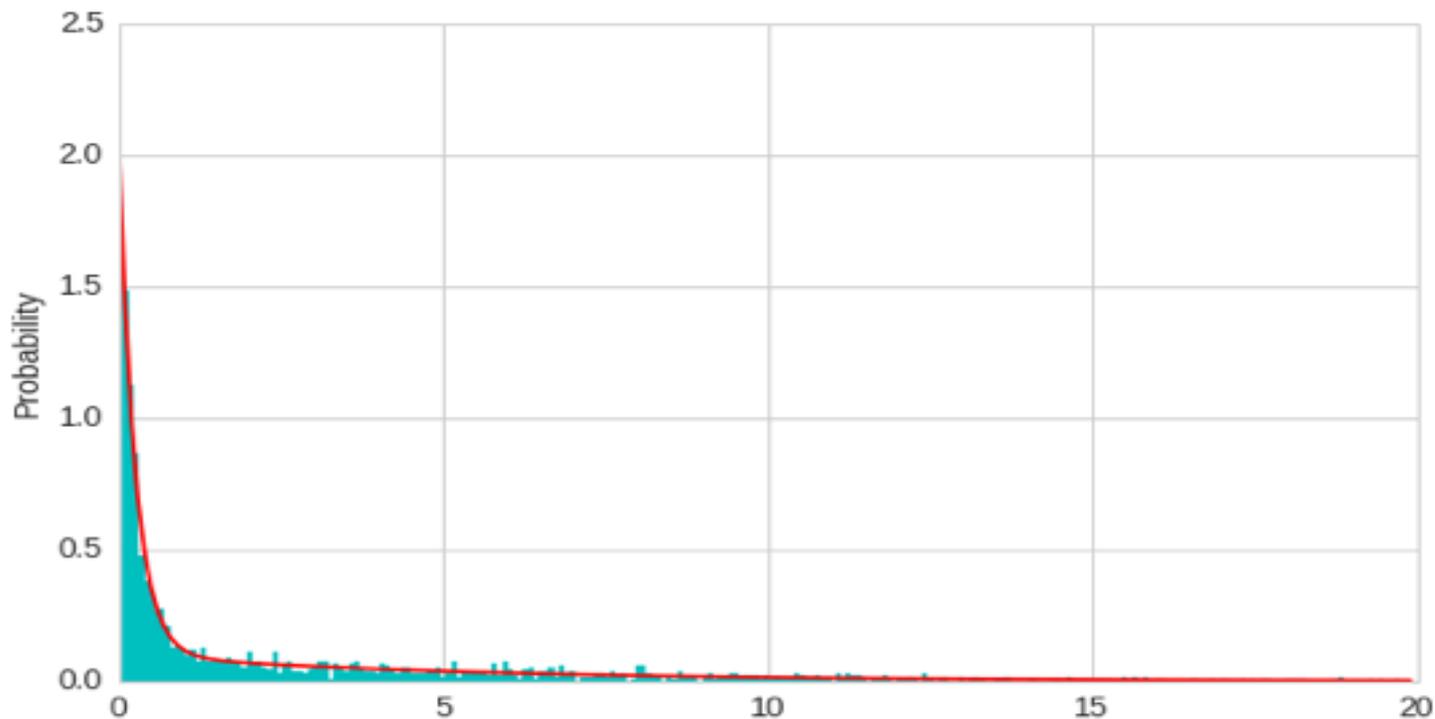
An exponential distribution is not right



model = ExponentialDistribution.from_samples(X)



A mixture of exponentials is better



```
model = GeneralMixtureModel.from_samples(ExponentialDistribution, 2, X)
```



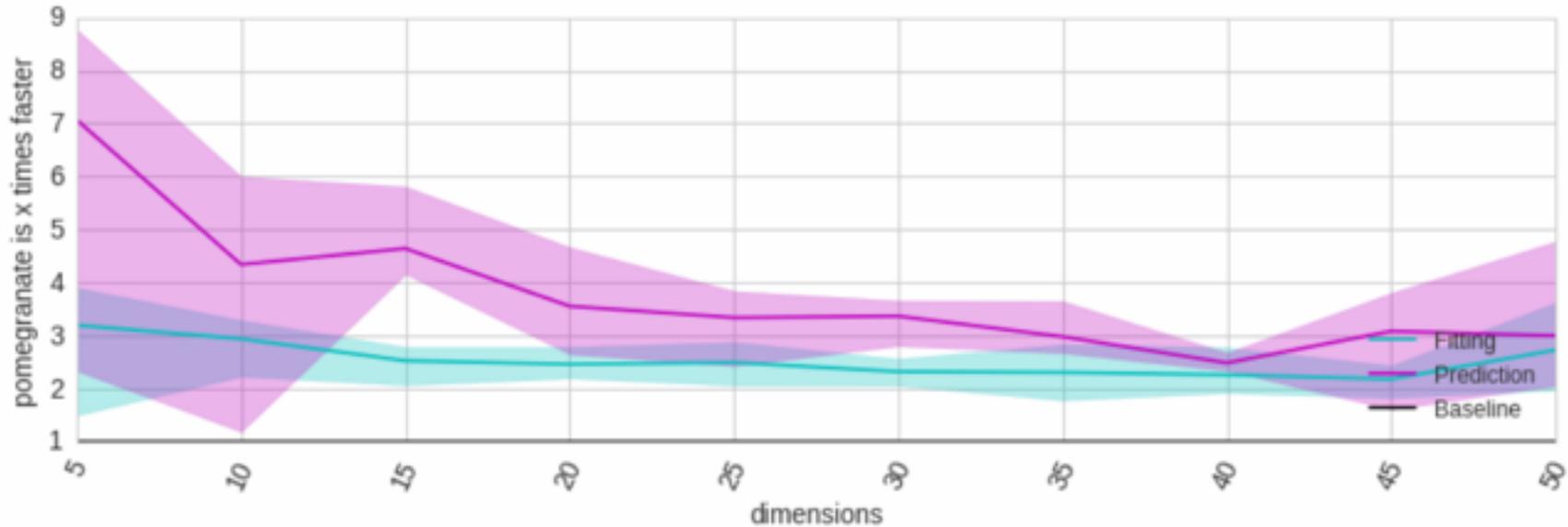
Heterogeneous mixtures natively supported



```
model = GeneralMixtureModel.from_samples([ExponentialDistribution, UniformDistribution], 2, X)
```



GMMs faster than sklearn





Overview: this talk

Overview

Major Models/Model Stacks

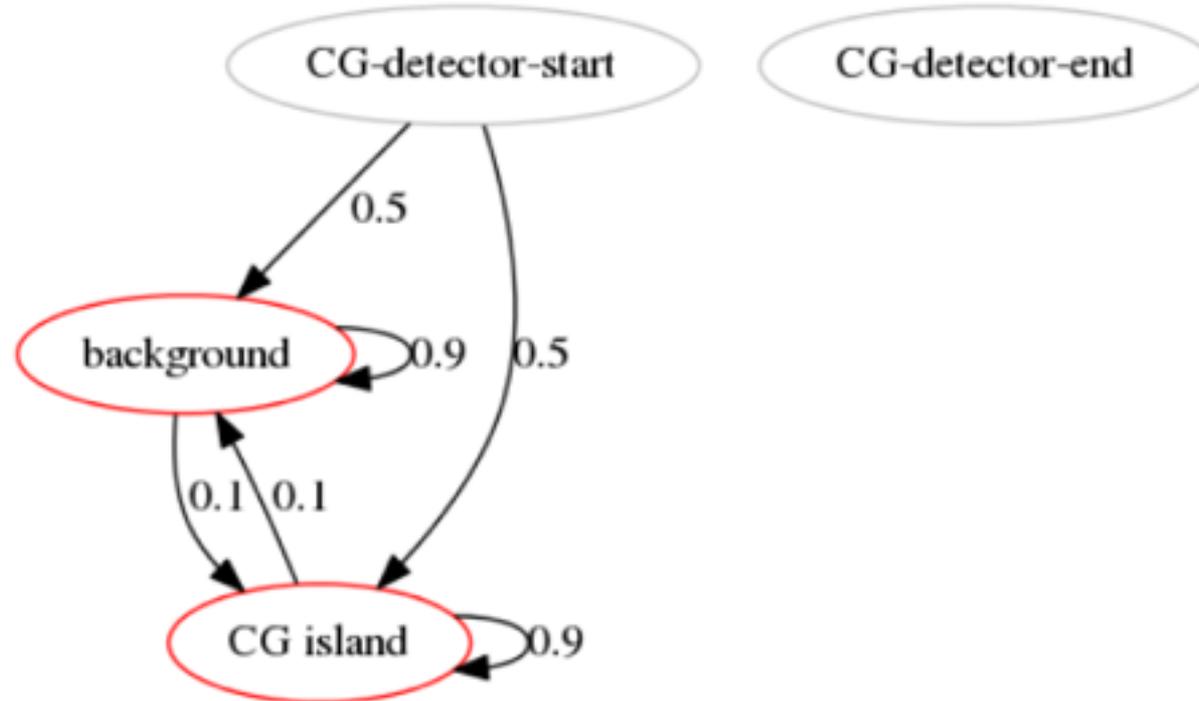
1. General Mixture Models
- 2. Hidden Markov Models**
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Finale: Train a mixture of HMMs in parallel



CG enrichment detection HMM

GACTACGACT**CGCGCTCGCACGTCGCTCG**ACATCATCGACA





CG enrichment detection HMM

GACTACGACTCGCGCTCGCACGTGCTCGACATCATCGACA

```
d1 = DiscreteDistribution({'A': 0.25, 'C': 0.25, 'G': 0.25, 'T': 0.25})
d2 = DiscreteDistribution({'A': 0.10, 'C': 0.40, 'G': 0.40, 'T': 0.10})

s1 = State(d1, name="background")
s2 = State(d2, name="CG island")

hmm = HiddenMarkovModel("CG-detector")
hmm.add_states(s1, s2)
hmm.add_transition(hmm.start, s1, 0.5)
hmm.add_transition(hmm.start, s2, 0.5)
hmm.add_transition(s1, s1, 0.9)
hmm.add_transition(s1, s2, 0.1)
hmm.add_transition(s2, s1, 0.1)
hmm.add_transition(s2, s2, 0.9)
hmm.bake()
```



pomegranate HMMs are feature rich

Feature	pomegranate	hmmlearn
Graph Structure		
Silent States	✓	
Optional Explicit End State	✓	
Sparse Implementation	✓	
Arbitrary Emissions Allowed on States	✓	
Discrete/Gaussian/GMM Emissions	✓	✓
Large Library of Other Emissions	✓	
Build Model from Matrices	✓	✓
Build Model Node-by-Node	✓	
Serialize to JSON	✓	
Serialize using Pickle/Joblib	✓	✓

Algorithms		
Priors		✓
Sampling	✓	✓
Log Probability Scoring	✓	✓
Forward-Backward Emissions	✓	✓
Forward-Backward Transitions	✓	
Viterbi Decoding	✓	✓
MAP Decoding	✓	✓
Baum-Welch Training	✓	✓
Viterbi Training	✓	
Labeled Training	✓	
Tied Emissions	✓	
Tied Transitions	✓	
Emission Inertia	✓	
Transition Inertia	✓	
Emission Freezing	✓	✓
Transition Freezing	✓	✓
Multi-threaded Training	✓	



GMM-HMM easy to define

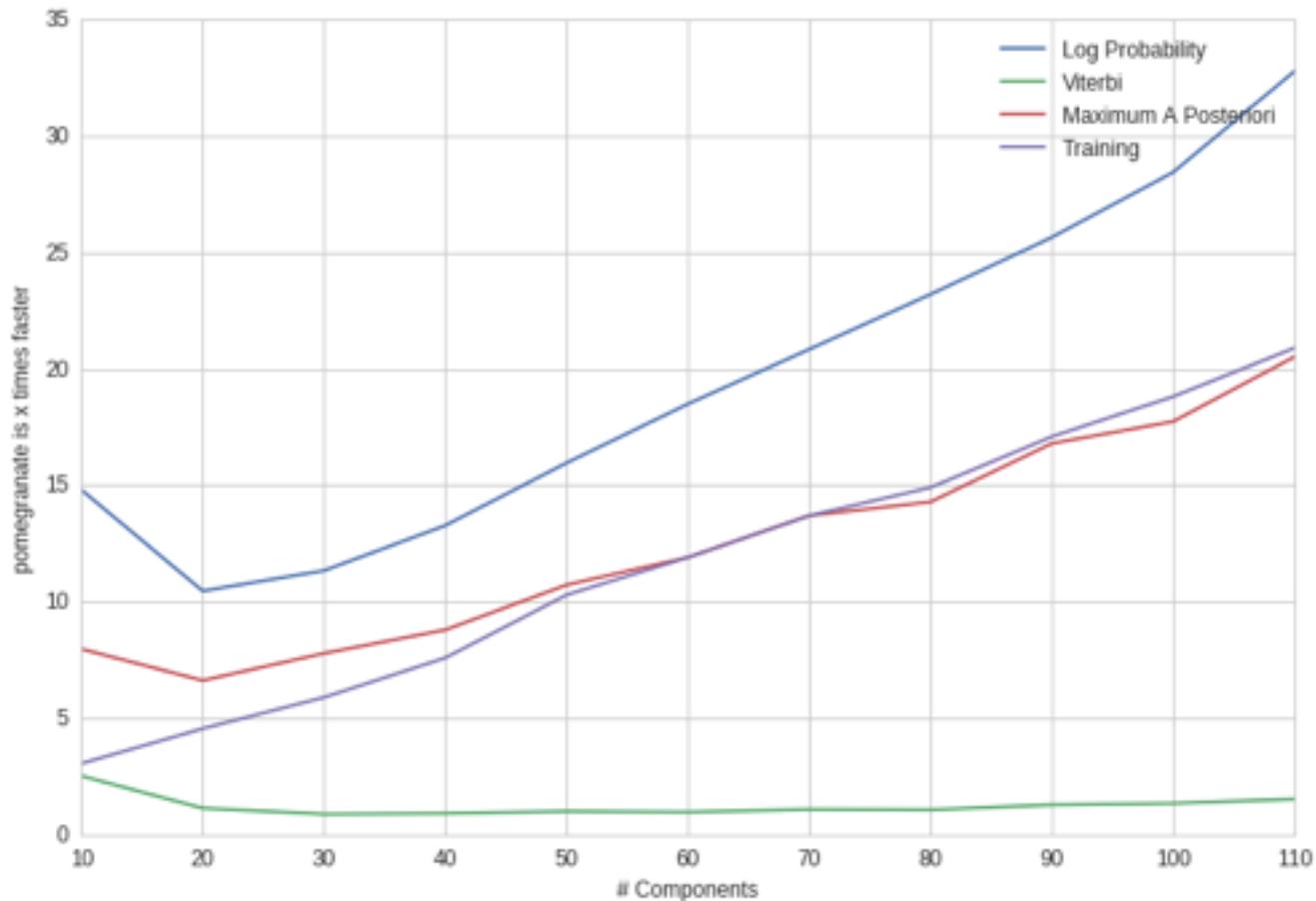
```
d1 = GeneralMixtureModel([NormalDistribution(5, 2), NormalDistribution(5, 4)])
d2 = GeneralMixtureModel([NormalDistribution(15, 1), NormalDistribution(15, 5)])

s1 = State(d1, name="GMM1")
s2 = State(d2, name="GMM2")

model = HiddenMarkovModel()
model.add_states(s1, s2)
model.add_transition(model.start, s1, 0.75)
model.add_transition(model.start, s2, 0.25)
model.add_transition(s1, s1, 0.85)
model.add_transition(s1, s2, 0.15)
model.add_transition(s2, s2, 0.90)
model.add_transition(s2, s1, 0.10)
model.bake()
```



HMMs are faster than hmmlearn





Overview: this talk

Overview

Major Models/Model Stacks

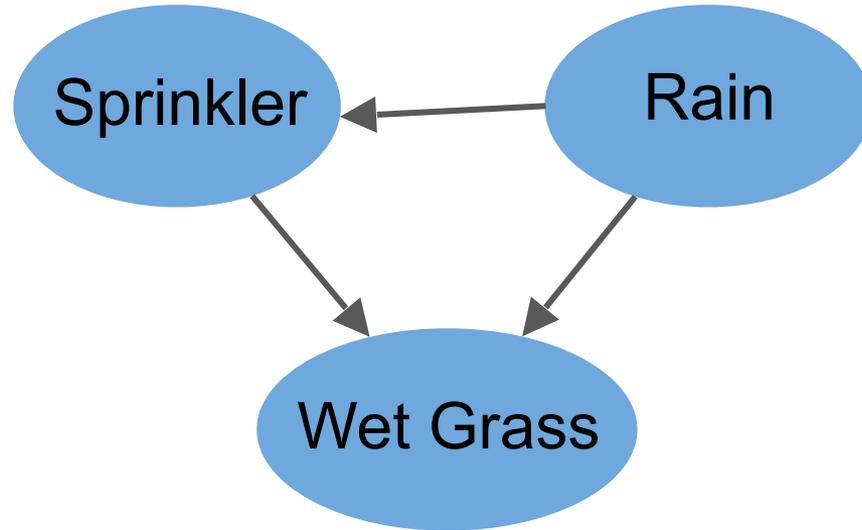
1. General Mixture Models
2. Hidden Markov Models
- 3. Bayesian Networks**
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Finale: Train a mixture of HMMs in parallel



Bayesian networks

Bayesian networks are powerful inference tools which define a dependency structure between variables.

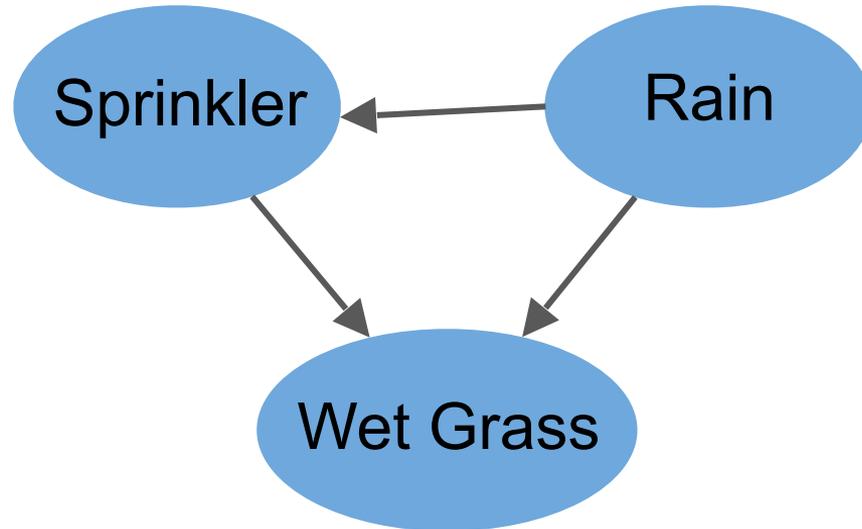




Bayesian networks

Two main difficult tasks:

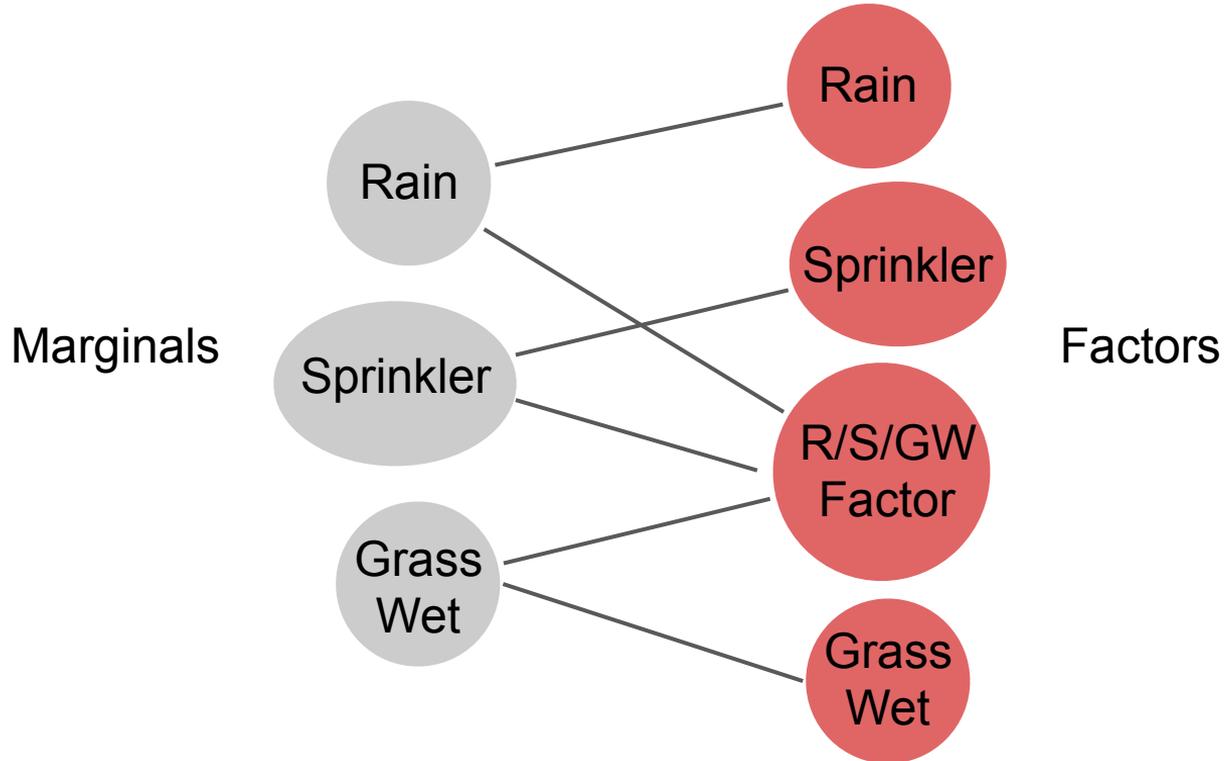
- (1) Inference given incomplete information
- (2) Learning the dependency structure from data





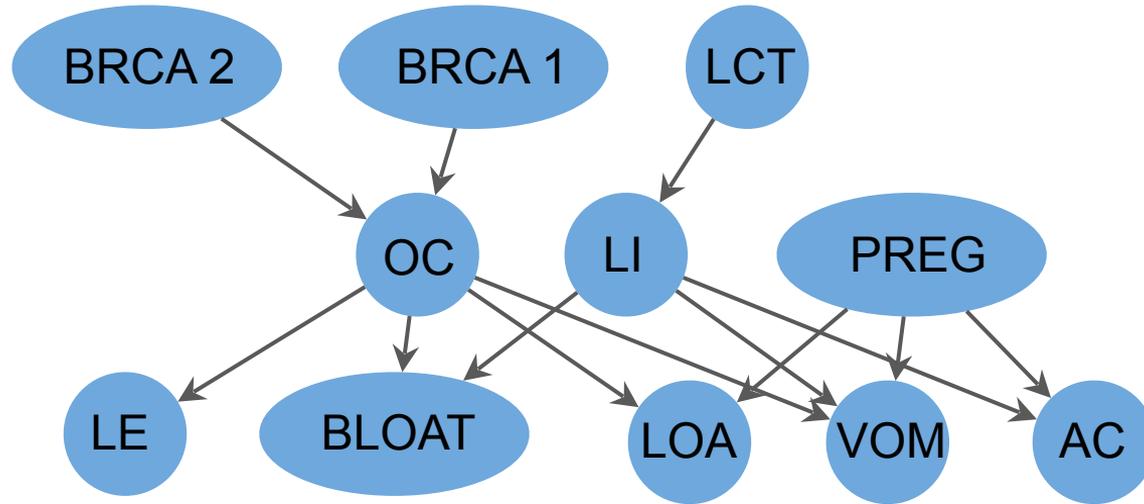
Bayesian network inference

Inference is done using Belief Propagation on a factor graph. Messages are sent from one side to the other until convergence.





Inference in a medical diagnostic network

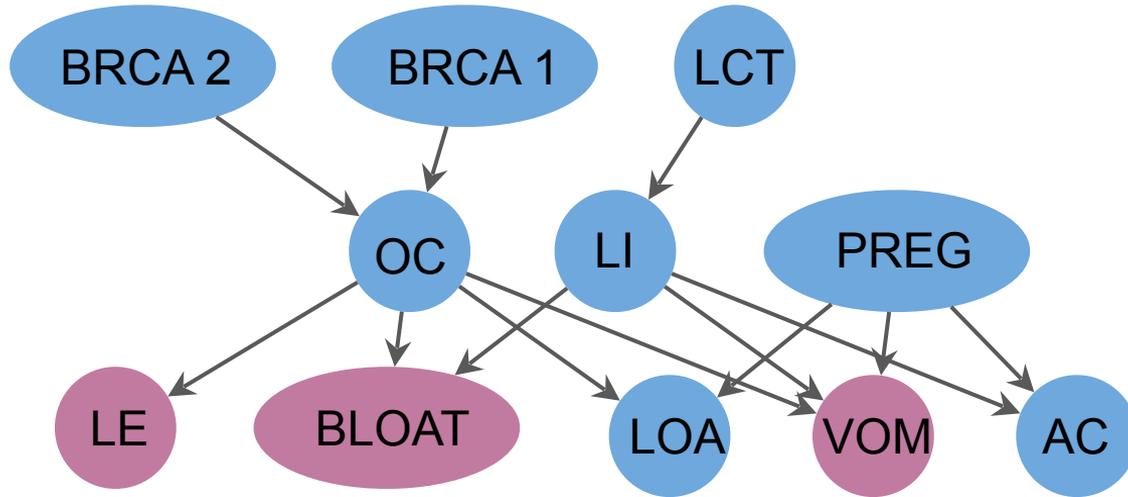


```
d = model.predict_proba()
print "\t".join( "{:7}".format(state.name) for state in model.states )
print "\t".join( "{:4.2}".format(model.parameters[0][1]) for model in d )
```

BRCA1	BRCA2	LCT	OC	LI	PREG	LE	BLOAT	LOA	VOM	AC
0.001	0.015	0.05	0.005	0.05	0.1	0.014	0.16	0.017	0.091	0.15



Inference in a medical diagnostic network



```
d = model.predict_proba({'VOM' : 1, 'BLOAT' : 1, 'LE' : 1})
print "\t".join( "{:7}".format(state.name) for state in model.states )
print "\t".join( "{:4.2}".format(model.parameters[0][1]) for model in d )
```

BRCA1	BRCA2	LCT	OC	LI	PREG	LE	BLOAT	LOA	VOM	AC
0.056	0.68	0.087	0.91	0.096	0.2	1.0	1.0	0.52	1.0	0.24



Bayesian network structure learning

???

Three primary ways:

- “Search and score” / Exact
- “Constraint Learning” / PC
- Heuristics



Bayesian network structure learning

???

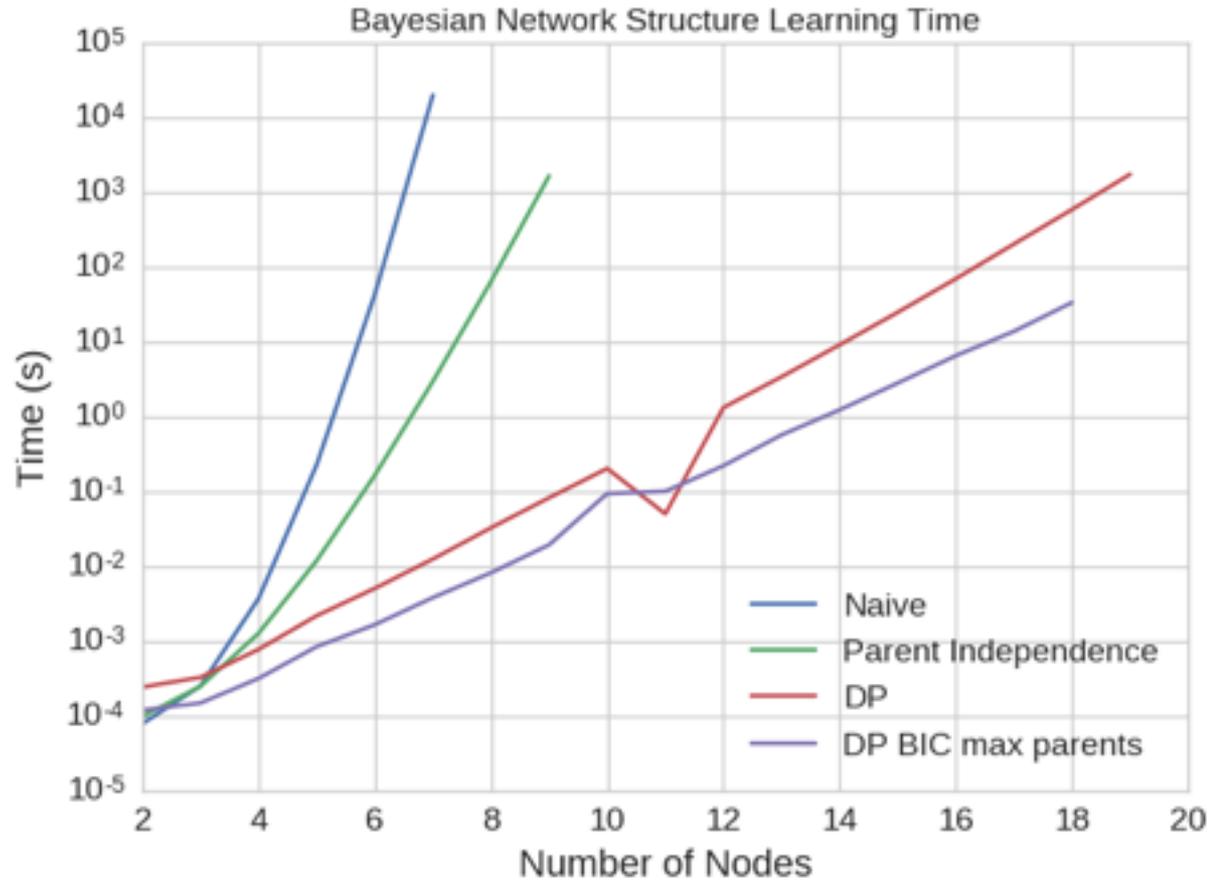
pomegranate supports:

- “Search and score” / Exact
- “Constraint Learning” / PC
- Heuristics



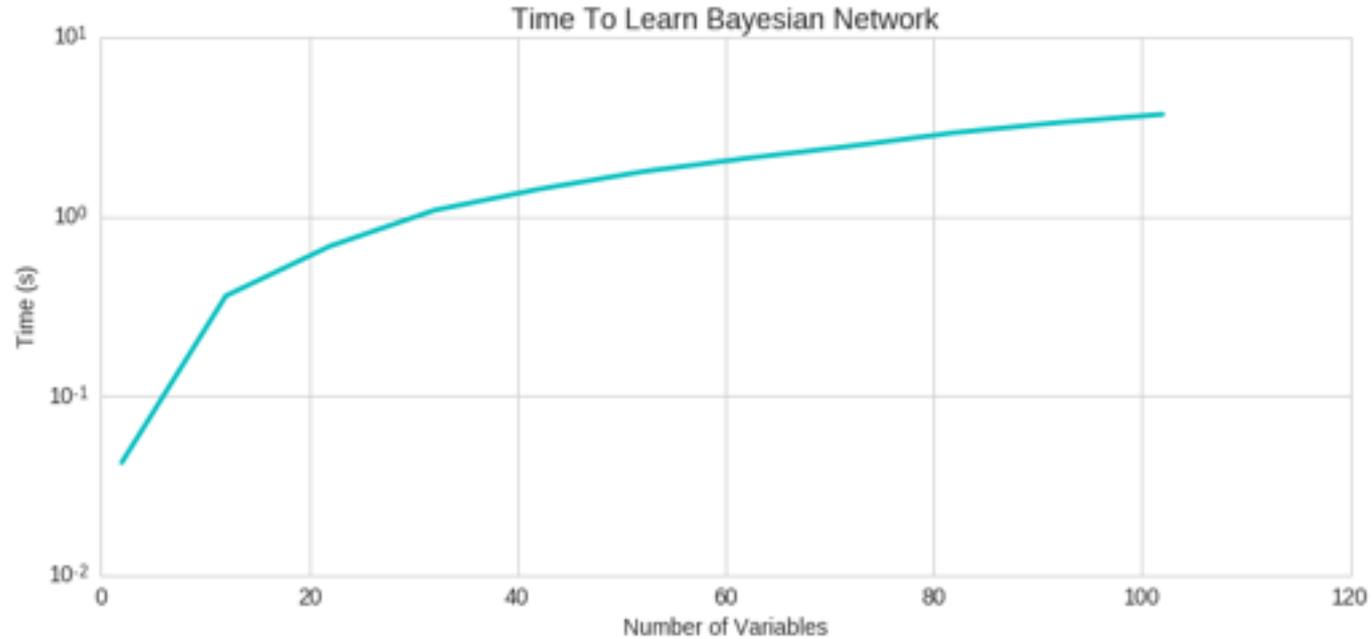
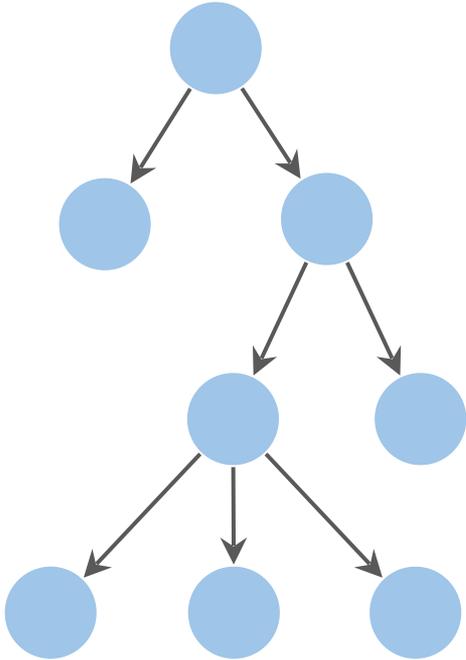
Exact structure learning is intractable

???





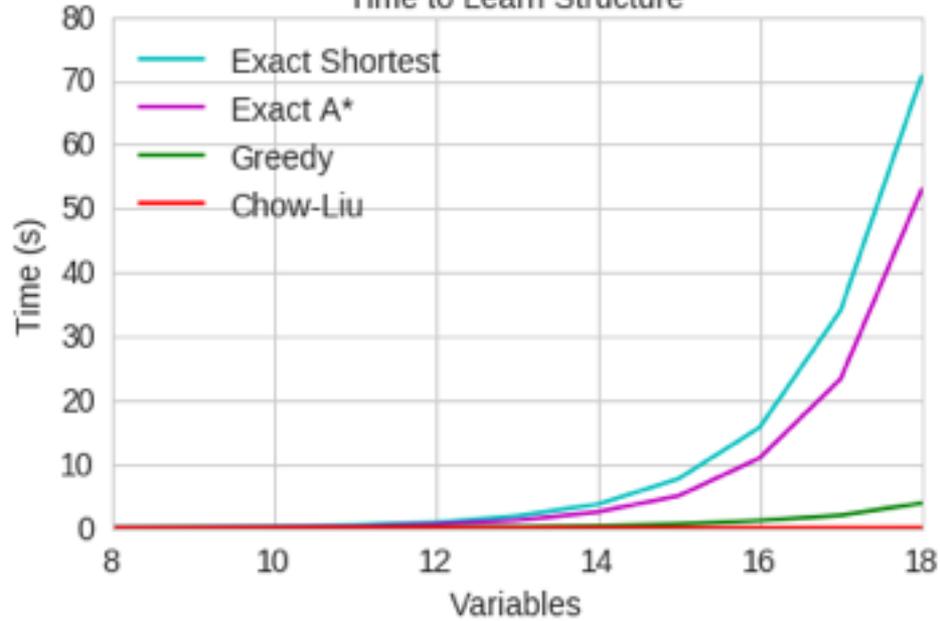
Chow-Liu trees are fast approximations



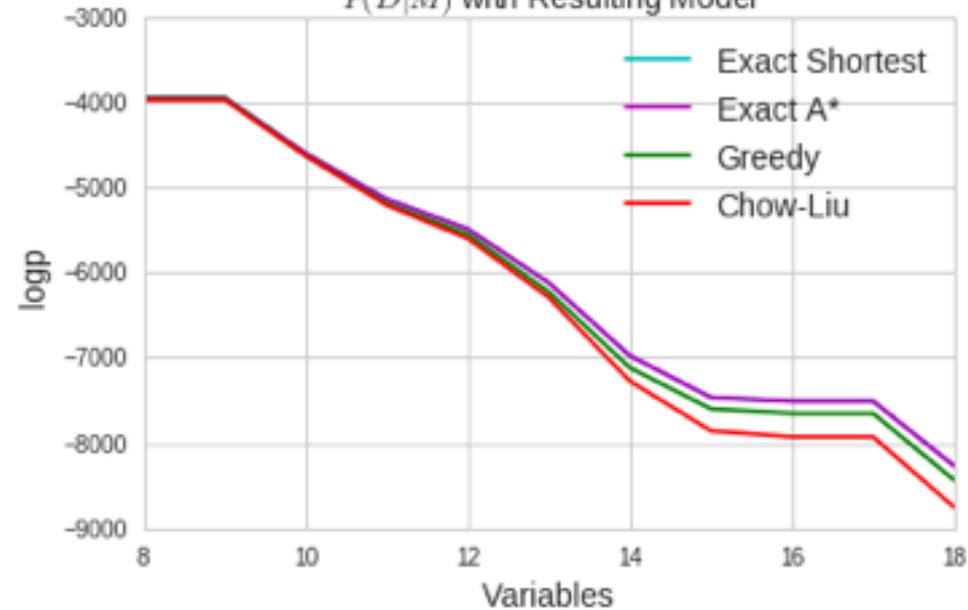


pomegranate supports four algorithms

Time to Learn Structure

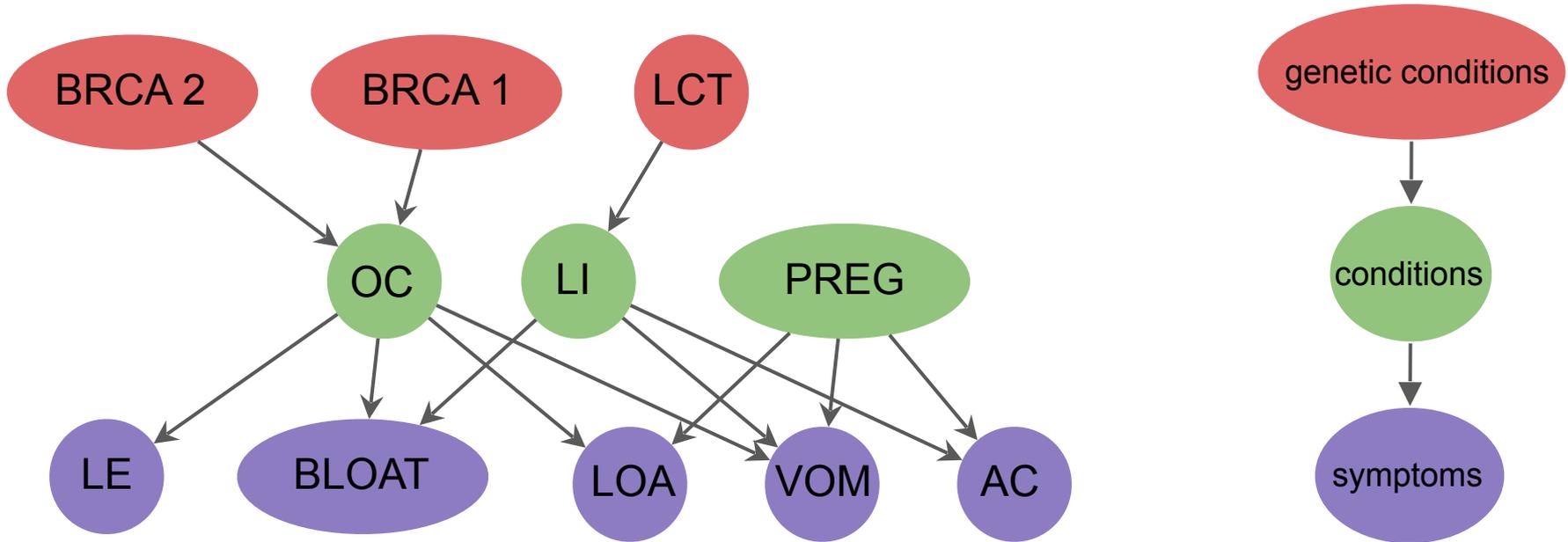


$P(D|M)$ with Resulting Model



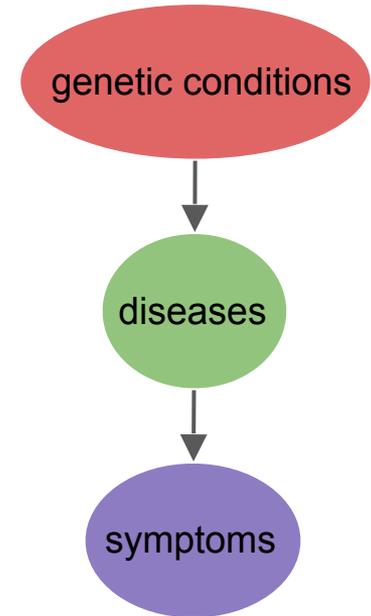
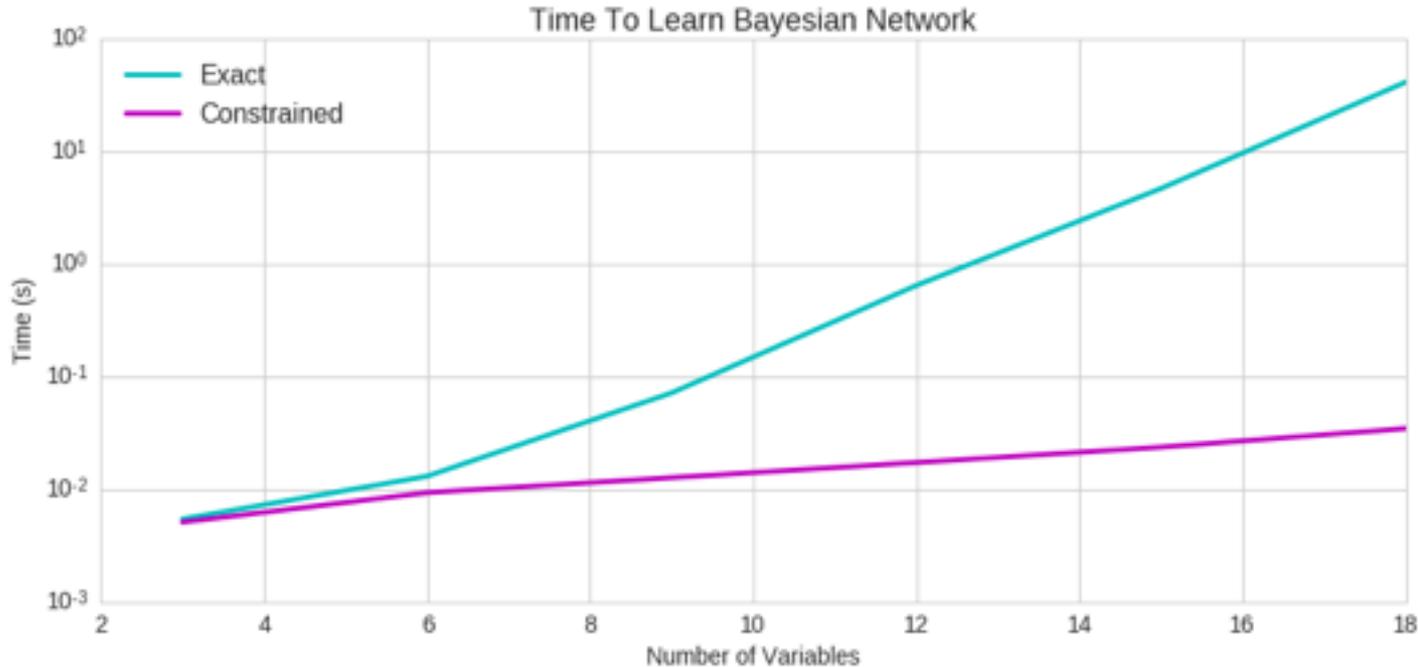


Constraint graphs merge data + knowledge



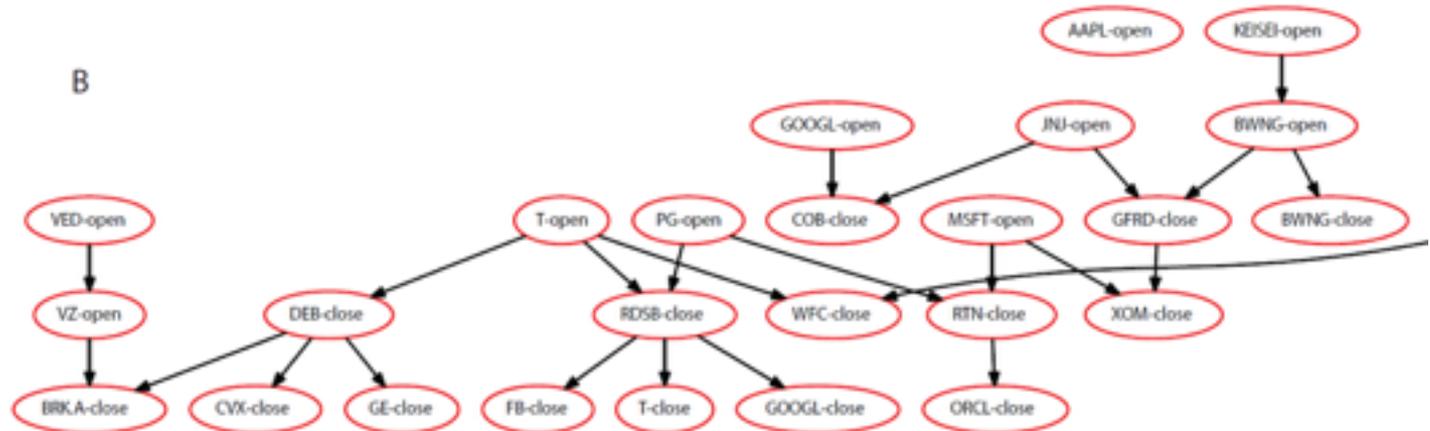
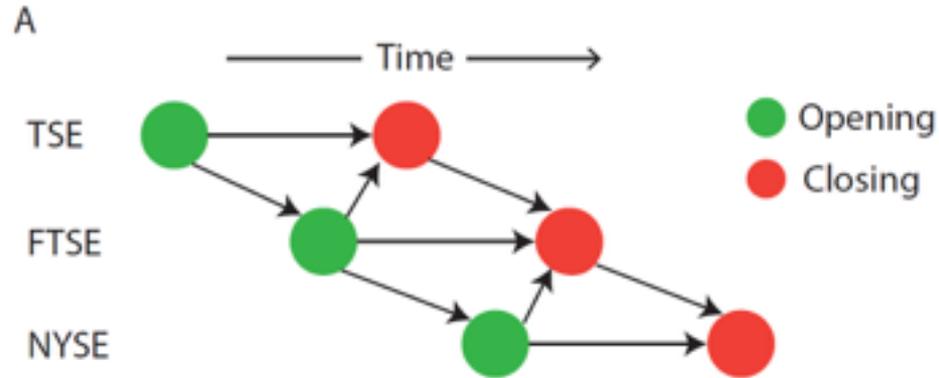


Constraint graphs merge data + knowledge





Modeling the global stock market





Finding the optimal Bayesian network given a constraint graph

Jacob M. Schreiber¹ and William S. Noble²

¹ Department of Computer Science, University of Washington, Seattle, WA, United States of America

² Department of Genome Science, University of Washington, Seattle, WA, United States of America

ABSTRACT

Despite recent algorithmic improvements, learning the optimal structure of a Bayesian network from data is typically infeasible past a few dozen variables. Fortunately, domain knowledge can frequently be exploited to achieve dramatic computational savings, and in many cases domain knowledge can even make structure learning tractable. Several methods have previously been described for representing this type of structural prior



Overview: this talk

Overview

Major Models/Model Stacks

1. General Mixture Models
2. Hidden Markov Models
3. Bayesian Networks
- 4. Bayes Classifiers**

Finale: Train a mixture of HMMs in parallel



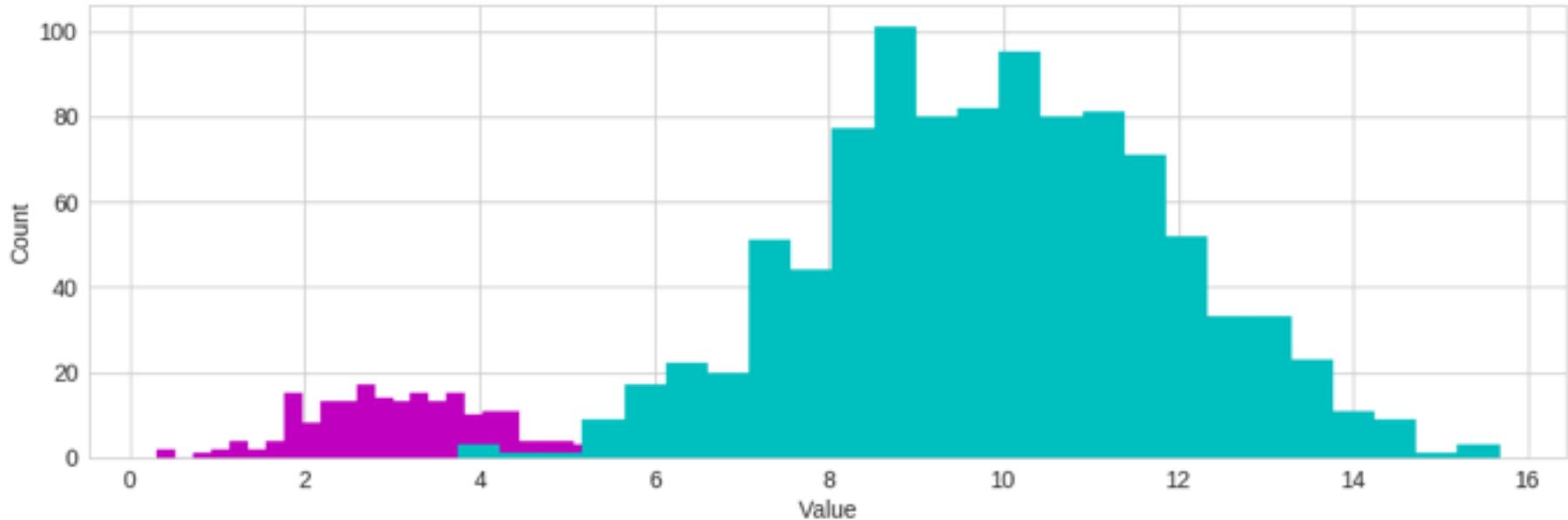
Bayes classifiers rely on Bayes' rule

$$P(M|D) = \frac{P(D|M)P(M)}{\sum_M P(D|M)P(M)}$$

$$\textit{Posterior} = \frac{\textit{Likelihood} * \textit{Prior}}{\textit{Normalization}}$$

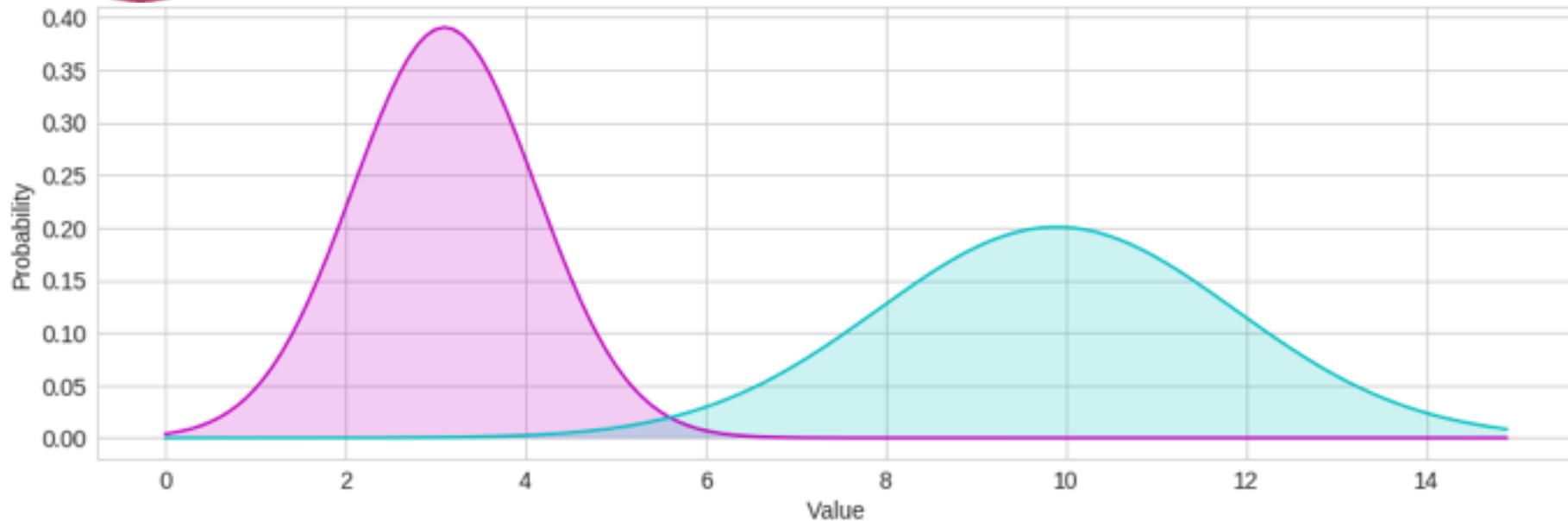


Let's build a classifier on this data



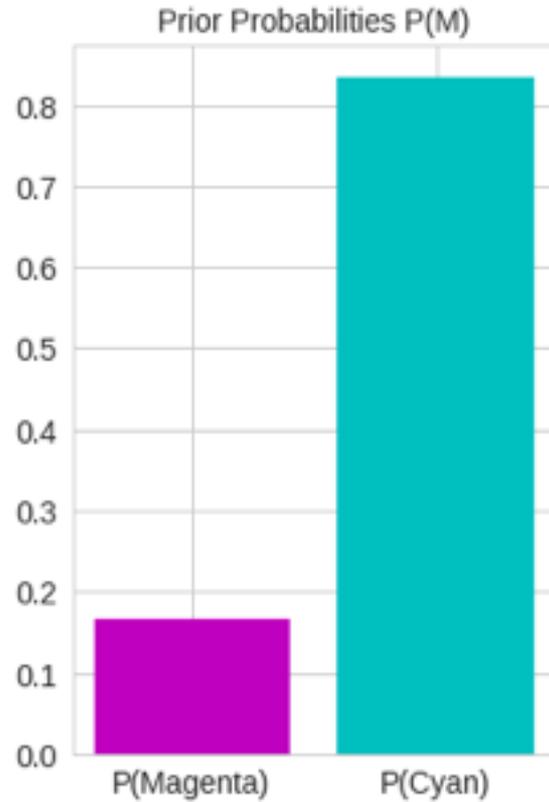


Likelihood function alone is not good



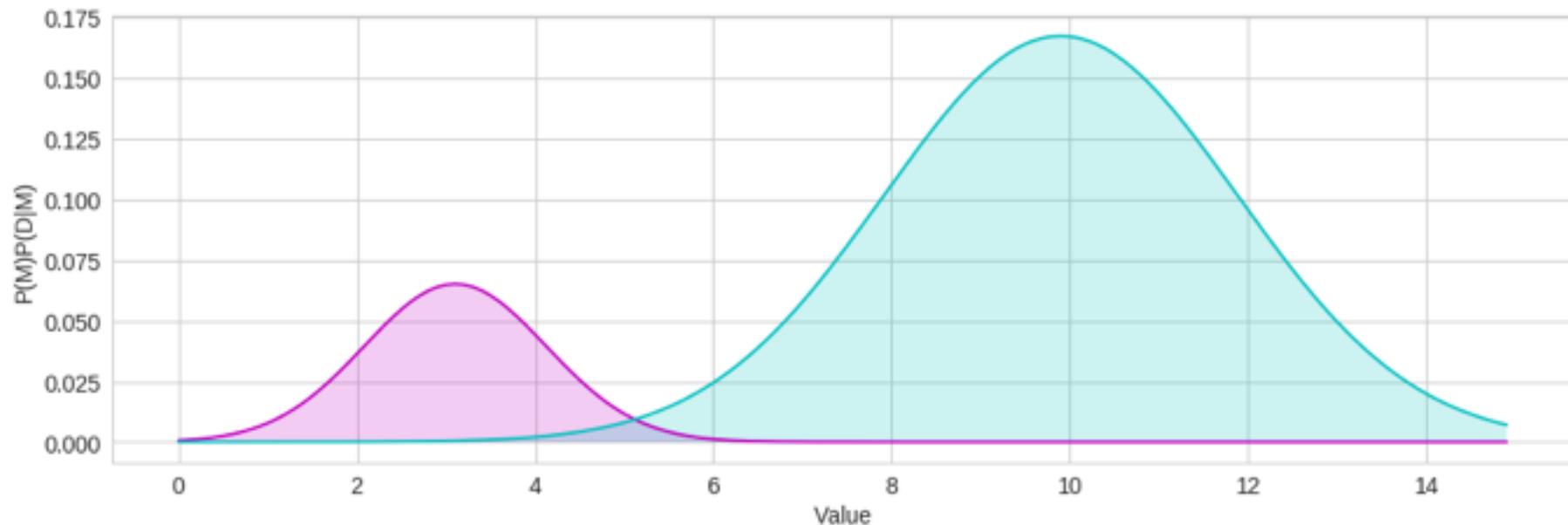


Priors can model class imbalance





The posterior is a good model of the data





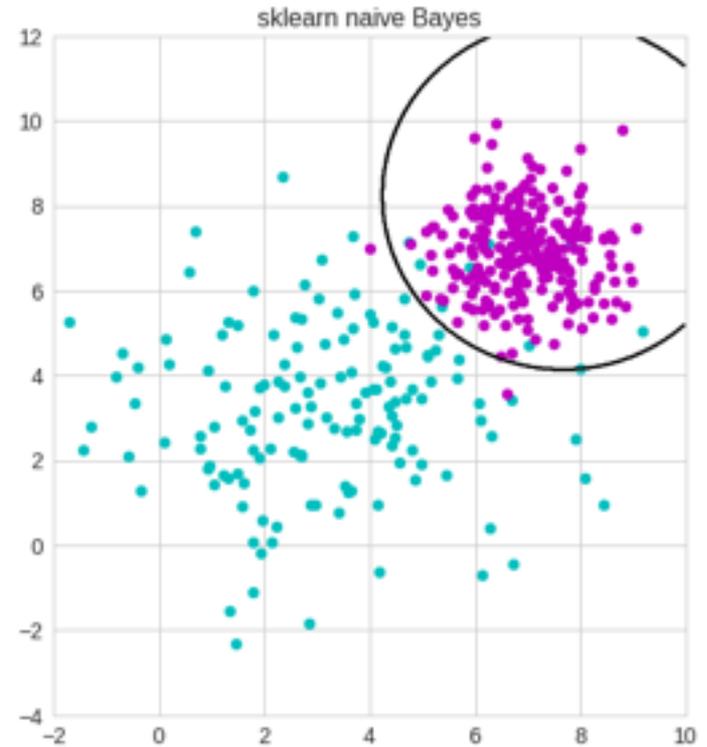
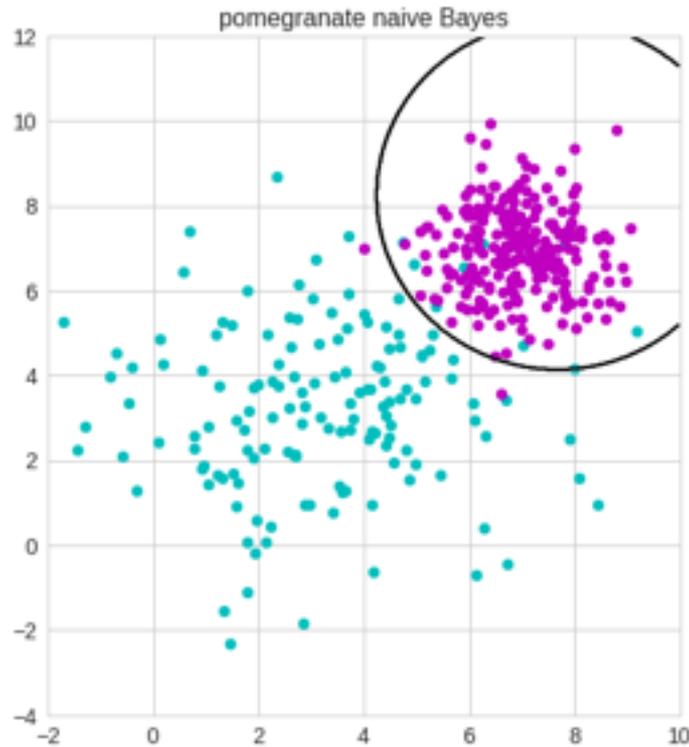
Naive Bayes assumes independent features

$$P(M|D) = \frac{\prod_{i=1}^d P(D_i|M)P(M)}{\sum_M \prod_{i=1}^d P(D_i|M)P(M)}$$

$$\textit{Posterior} = \frac{\textit{Likelihood} * \textit{Prior}}{\textit{Normalization}}$$



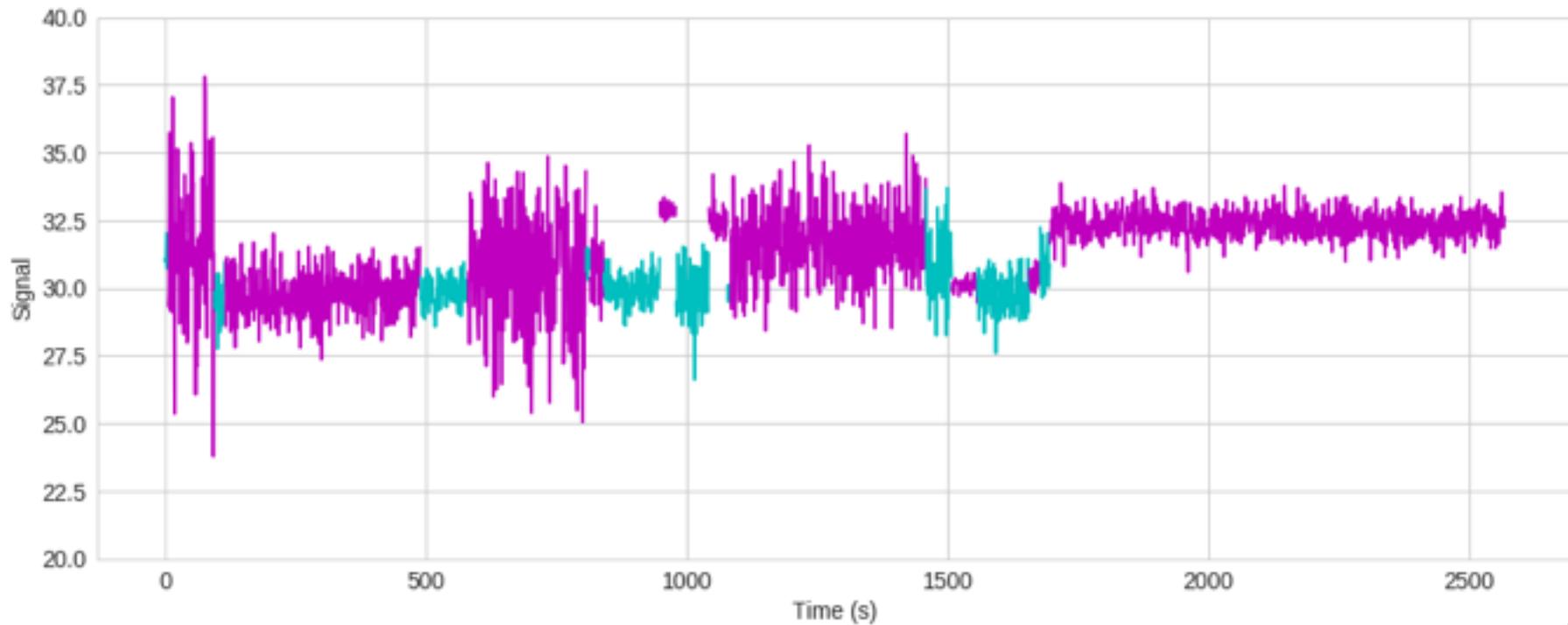
Naive Bayes produces ellipsoid boundaries



`model = NaiveBayes.from_samples(NormalDistribution, X, y)`



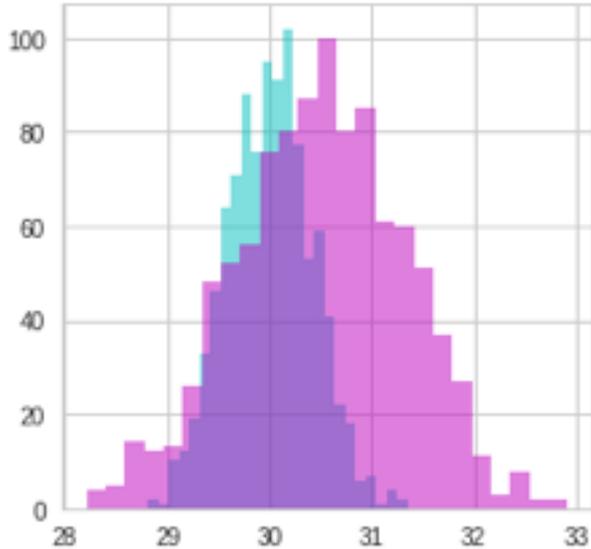
Naive Bayes can be heterogeneous



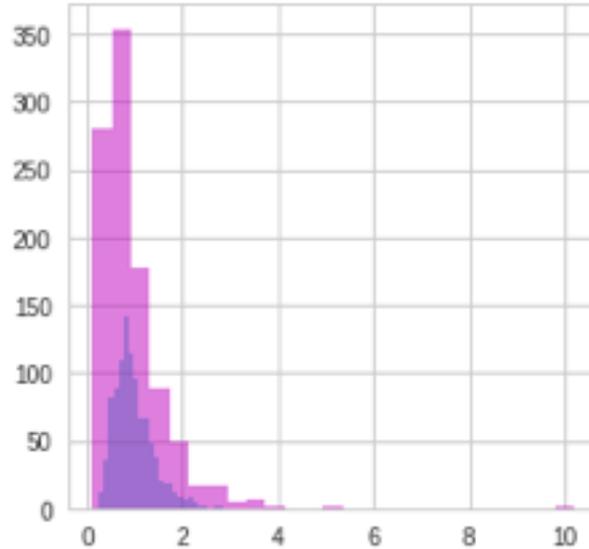


Data can fall under different distributions

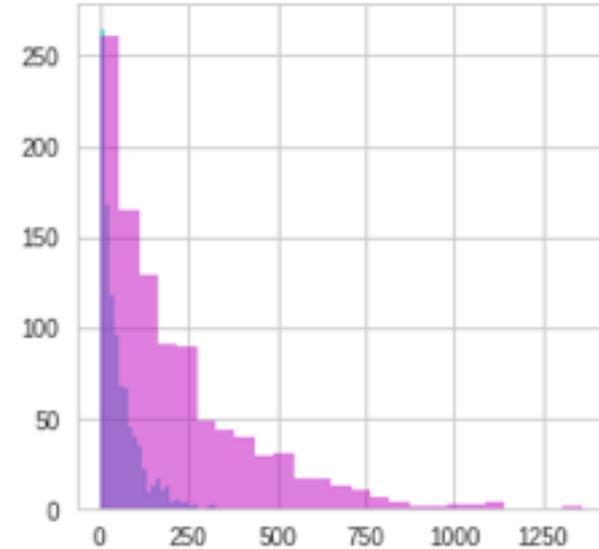
Mean



Standard Deviation



Duration





Using appropriate distributions is better

```
model = NaiveBayes.from_samples(NormalDistribution, X_train, y_train)
```

```
print "Gaussian Naive Bayes: ", (model.predict(X_test) == y_test).mean()
```

```
clf = GaussianNB().fit(X_train, y_train)
```

```
print "sklearn Gaussian Naive Bayes: ", (clf.predict(X_test) == y_test).mean()
```

```
model = NaiveBayes.from_samples([NormalDistribution, LogNormalDistribution,  
ExponentialDistribution], X_train, y_train)
```

```
print "Heterogeneous Naive Bayes: ", (model.predict(X_test) == y_test).mean()
```

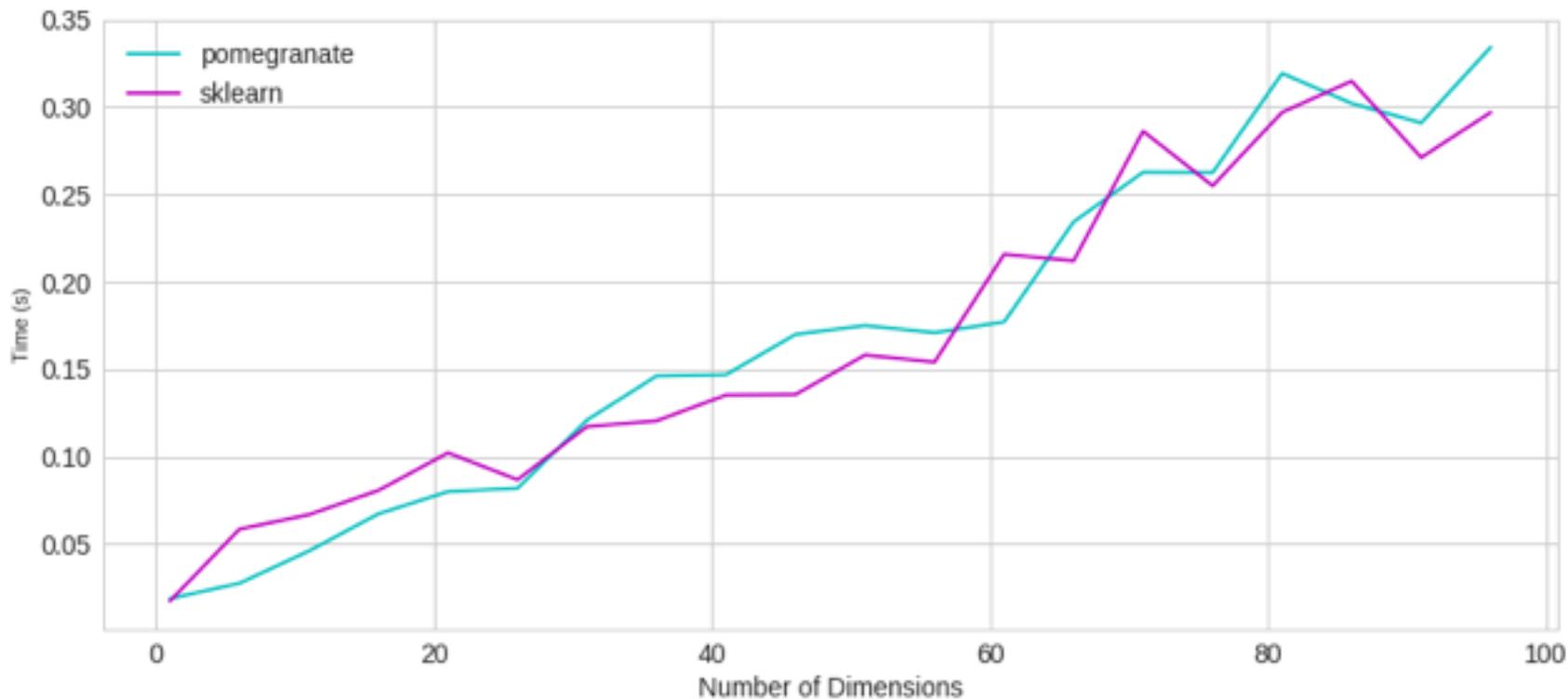
Gaussian Naive Bayes: 0.798

sklearn Gaussian Naive Bayes: 0.798

Heterogeneous Naive Bayes: 0.844



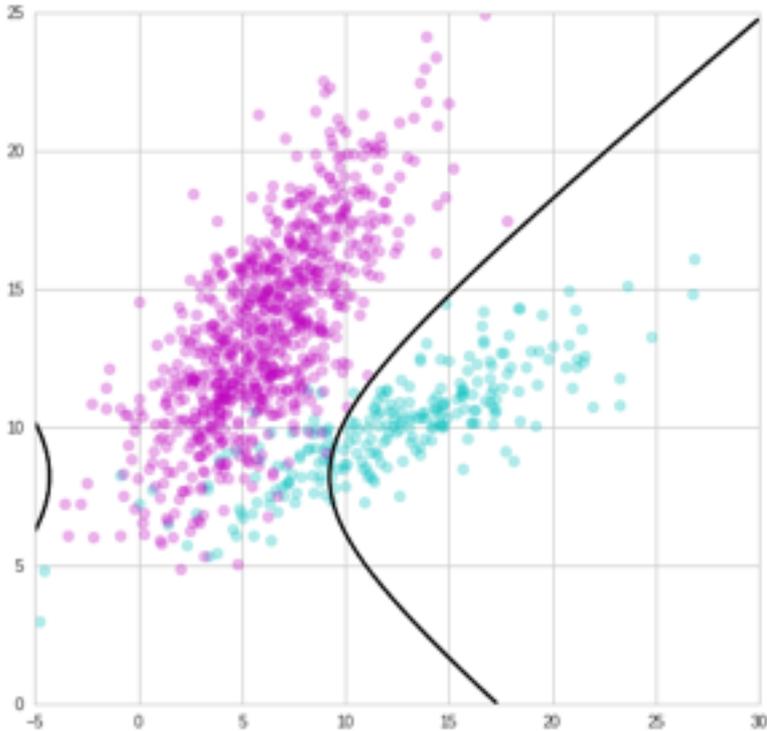
This additional flexibility is just as fast



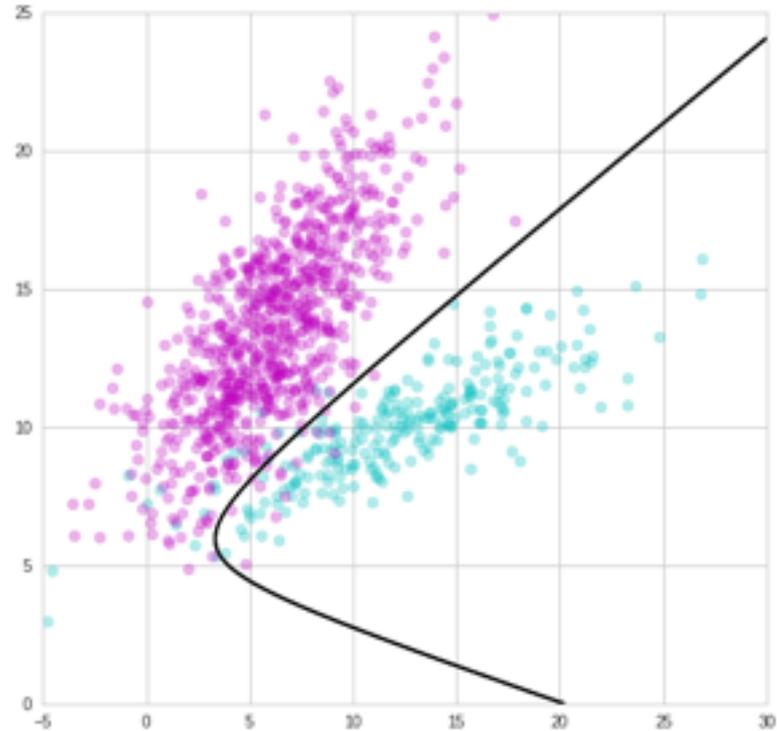


Bayes classifiers don't require independence

naive accuracy: 0.929

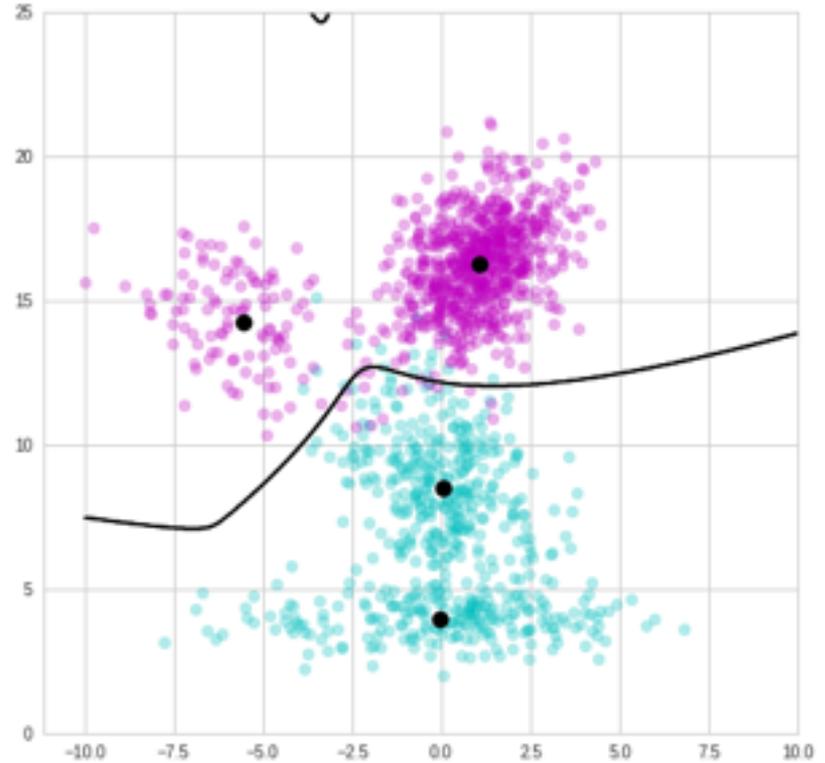
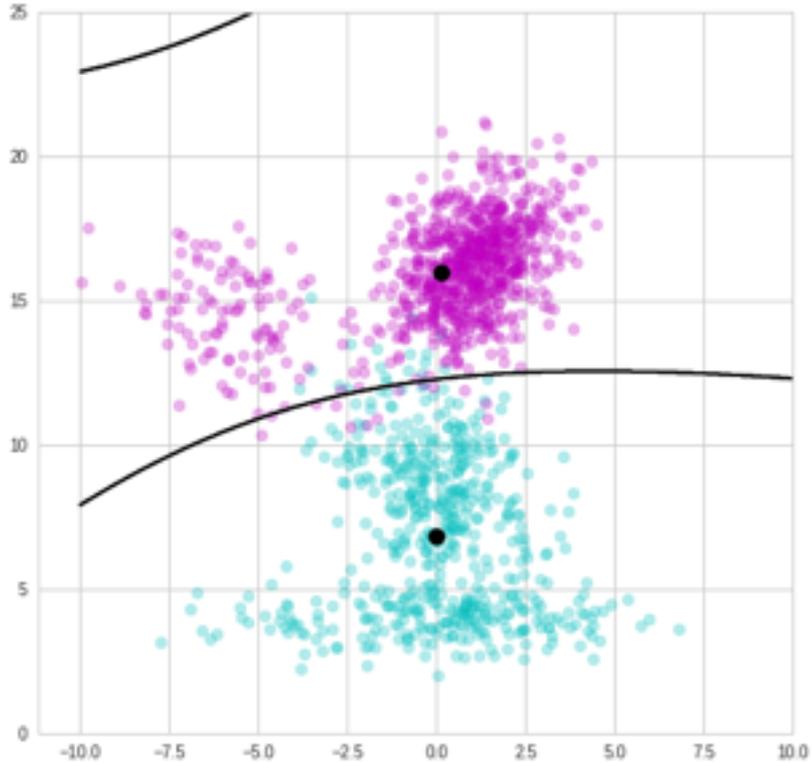


bayes classifier accuracy: 0.966





Gaussian mixture model Bayes classifier





Creating complex Bayes classifiers is easy

```
gmm_a = GeneralMixtureModel.from_samples(MultivariateGaussianDistribution, 2, X[y == 0])  
gmm_b = GeneralMixtureModel.from_samples(MultivariateGaussianDistribution, 2, X[y == 1])  
model_b = BayesClassifier([gmm_a, gmm_b], weights=numpy.array([1-y.mean(), y.mean()]))
```



Creating complex Bayes classifiers is easy

```
mc_a = MarkovChain.from_samples(X[y == 0])
mc_b = MarkovChain.from_samples(X[y == 1])
model_b = BayesClassifier([mc_a, mc_b], weights=numpy.array([1-y.mean(), y.mean()]))
```

```
hmm_a = HiddenMarkovModel.from_samples(X[y == 0])
hmm_b = HiddenMarkovModel.from_samples(X[y == 1])
model_b = BayesClassifier([hmm_a, hmm_b], weights=numpy.array([1-y.mean(), y.mean()]))
```

```
bn_a = BayesianNetwork.from_samples(X[y == 0])
bn_b = BayesianNetwork.from_samples(X[y == 1])
model_b = BayesClassifier([bn_a, bn_b], weights=numpy.array([1-y.mean(), y.mean()]))
```



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Training a mixture of HMMs in parallel

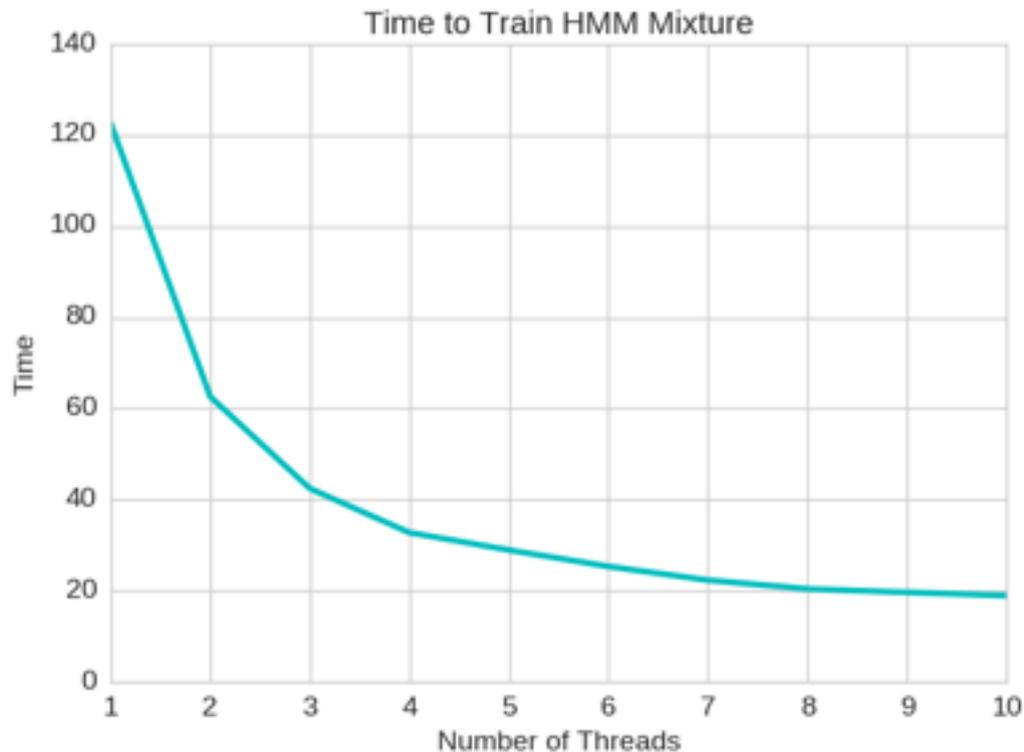
Creating a mixture of HMMs is just as simple as passing the HMMs into a GMM as if it were any other distribution

```
model_C = create_profile_hmm(dC, I)
model_mC = create_profile_hmm(dmC, I)
model_hmC = create_profile_hmm(dhmC, I)

model = GeneralMixtureModel([model_C, model_mC, model_hmC])
return model
```



Training a mixture of HMMs in parallel



`fit(model, X, n_jobs=n)`



Documentation available at Readthedocs

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Home

pomegranate is a python package which implements fast, efficient, and extremely flexible probabilistic models ranging from probability distributions to Bayesian networks to mixtures of hidden Markov models. The most basic level of probabilistic modeling is the a simple probability distribution. If we're modeling language, this may be a simple distribution over the frequency of all possible words a person can say.



Tutorials available on github

Branch: master | pomegranate / tutorials /

Create new file | Upload files | Find file | History

jmschrei ADD bayes backend | Latest commit 724510d 10 hours ago

..		
GGBlasts.xlsx	PyData Chicago 2016	8 months ago
PyData_2016_Chicago_Tutorial.ipynb	FIX markov chain notebooks	3 months ago
README.md	Update README.md	2 years ago
Tutorial_0_pomegranate_overview.ipynb	Minor typos	3 months ago
Tutorial_1_Distributions.ipynb	ENH tutorials	2 years ago
Tutorial_2_General_Mixture_Models.ipynb	FIX hmm dimensionality	11 months ago
Tutorial_3_Hidden_Markov_Models.ipynb	edit tutorial 3 to remove deprecated bake	7 months ago
Tutorial_4_Bayesian_Networks.ipynb	ENH pomegranate vs libpgm tutorial	7 months ago
Tutorial_4b_Bayesian_Network_Structure_Learning.i...	ENH a* search	28 days ago
Tutorial_5_Bayes_Classifiers.ipynb	ADD bayes backend	10 hours ago
Tutorial_6_Markov_Chain.ipynb	FIX markov chain notebooks	3 months ago
Tutorial_7_Parallelization.ipynb	ADD tutorial 7 parallelization	8 months ago

<https://github.com/jmschrei/pomegranate/tree/master/tutorials>



Acknowledgements



UNIVERSITY of WASHINGTON

eScience Institute

ADVANCING DATA-INTENSIVE DISCOVERY IN ALL FIELDS

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 PARIETAL

 **aspentech**

pomegranate

fast and flexible probabilistic modeling in python

Maxwell Libbrecht

Postdoc, Department of Genome Sciences,
University of Washington, Seattle WA
Assistant Professor, Simon Fraser University,
Vancouver BC

noble.gs.washington.edu/~maxwl/

Jacob Schreiber

PhD student, Paul G. Allen School of
Computer Science, University of
Washington, Seattle WA



[jmschreiber91](#)



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