Assume two rides with respective Normalized Power and duration t

Ride 1 NP_1, t_1

Ride 2 NP_2, t_2

The combination of the two yields a ride with

$$NP_{\text{tot}} = \left(\frac{NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2}{t_1 + t_2} \right)^{1/4}$$

$$t_{\text{tot}} = t_1 + t_2$$

Starting from the TSS formula and letting the exponents as variable, we have

$$cTSS = \left(\frac{NP}{FTP} \right)^x \cdot t^y$$

where multiplication factors 3600 and 100 are left out (they can be included in the units of t).

If we want additivity, we need to have

$$cTSS_{\text{tot}} = cTSS_1 + cTSS_2$$

$$\frac{1}{FTP^x} \cdot \left(\frac{NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2}{t_1 + t_2} \right)^{x/4} \cdot (t_1 + t_2)^y = \left(\frac{NP_1}{FTP} \right)^x \cdot t_1^y + \left(\frac{NP_2}{FTP} \right)^x \cdot t_2^y \qquad \text{simplify FTP}$$

$$\left(\frac{NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2}{t_1 + t_2} \right)^{x/4} \cdot (t_1 + t_2)^y = NP_1^x \cdot t_1^y + NP_2^x \cdot t_2^y \qquad \text{divide by } (t_1 + t_2)^y$$

$$\left(\frac{NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2}{t_1 + t_2} \right)^{x/4} = NP_1^x \cdot \left(\frac{t_1}{t_1 + t_2} \right)^y + NP_2^x \cdot \left(\frac{t_1}{t_1 + t_2} \right)^y$$

Solving this for the general case (all sets of NP_1, NP_2, t_1, t_2) is what we want. But it should be true for all specific case, so let's take a corner case, $NP_2 = 0$, which leads to

$$NP_1^x \left(\frac{t_1}{t_1 + t_2} \right)^{x/4} = NP_1^x \left(\frac{t_1}{t_1 + t_2} \right)^y$$

or

$$x = 4y$$

Inserting this into the general case

$$\left(\frac{NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2}{t_1 + t_2} \right)^y = NP_1^{4y} \cdot \left(\frac{t_1}{t_1 + t_2} \right)^y + NP_2^{4y} \cdot \left(\frac{t_1}{t_1 + t_2} \right)^y$$

$$\left(NP_1^4 \cdot t_1 + NP_2^4 \cdot t_2 \right)^y = \left(NP_1^4 \cdot t_1 \right)^y + \left(NP_2^4 \cdot t_2 \right)^y$$

which is of the form $(a + b)^y = a^y + b^y$ and only true for $y = 1$.

In conclusion,

$$cTSS = \left(\frac{NP}{FTP} \right)^4 \cdot t$$