(*

$$\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q})] = \frac{\pi}{2}\sum_i \frac{\omega_i{}^2 f_i}{\omega_i(q)}\,\delta(\omega-\omega_i(\mathbf{q}))$$

Now if

$$f_i = \frac{2}{\pi\omega_i^2}\int_{\omega_i-\Delta/2}^{\omega_i+\Delta/2}\omega\,\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q}=0)]\,d\omega$$

then

$$\int_0^\infty \omega\,\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q})]\,d\omega =$$

$$\frac{\pi}{2}\sum_i \omega_i{}^2 f_i =$$

$$\sum_i \int_{\omega_i-\Delta/2}^{\omega_i+\Delta/2}\omega\,\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q}=0)]\,d\omega$$

$$= \int_0^{\omega_{max}}\omega\,\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q}=0)]\,d\omega$$

This formula automatically satisfies the partial sum rules of $\mathrm{Im}[\epsilon^{-1}(\omega)]$

$$\int_0^\infty \omega\,\mathrm{Im}[\epsilon^{-1}(\omega,\mathbf{q})]\,d\omega = \frac{\pi}{2}\,\omega_p{}^2 N_{eff}$$

With this model,
the self energy formula should be
$\Sigma(E,k) = \sum_i f_i\,\Sigma_i(E,k)$
where
$\Sigma_i(E,k)$ **is just the HL self energy**
with ω_i replacing ω_p everywhere.

*)